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The impact of uncertainty and certainty shocks

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Non-technical summary

Research question

Major economic and political shocks, such as Black Monday or the 9/11 terrorist attacks, dramatically increase uncertainty. A series of studies shows that these sudden increases in uncertainty have negative real economic effects. For that reason, uncertainty shocks are an ubiquitous concern for policy makers. This study aims to broaden our understanding of uncertainty shocks building on a novel econometric approach.

Contribution

My econometric approach can distinguish the impact of sudden increases in uncertainty (uncertainty shocks) from sudden reductions in uncertainty (certainty shocks). Commonly employed methods do not separate the two types of shocks. At the same time, the approach serves to analyze the impact of uncertainty shocks on the downside and upside risks to the real economy. Such analysis provides additional information for policy makers. It informs about the potential scenarios that may materialize after an uncertainty shock.

Results

I find that it is important to distinguish uncertainty shocks from certainty shocks. For instance, the impact of an uncertainty shock becomes stronger if separated from a certainty shock. In parallel, I show that an uncertainty shock increases both the downside and the upside risks to the real economy. However, the downside risks increase much more strongly.

Nichttechnische Zusammenfassung

Fragestellung

Schwere ökonomische und politische Schocks wie der Schwarze Montag oder die Terroranschläge vom 11. September 2001 führen zu erheblicher Unsicherheit. Eine Reihe von Studien zeigt, dass ein abrupter Anstieg der Unsicherheit mit negativen Folgen für die Realwirtschaft verbunden ist. Daher sind Unsicherheitsschocks ein allgegenwärtiges Thema für wirtschaftspolitische Entscheidungsträger. Ziel dieser Studie ist es, mithilfe einer neuen ökonometrischen Methode zu einem besseren Verständnis von Unsicherheitsschocks zu gelangen.

Beitrag

Während die gängigen Methoden nicht zwischen den Auswirkungen eines plötzlichen Anstiegs (Unsicherheitsschock) und eines plötzlichen Rückgangs (Sicherheitsschock) von Unsicherheit differenzieren, kann die von mir vorgeschlagene ökonometrische Methode diese Unterscheidung vornehmen. Gleichzeitig eignet sie sich, die Effekte von Unsicherheitsschocks auf Abwärts- und Aufwärtsrisiken der Realwirtschaft herauszuarbeiten und liefert somit zusätzliche Informationen für Entscheidungsträger. Die Analyse gibt Auskunft über mögliche Szenarios, die nach einem Unsicherheitsschock eintreten können.

Ergebnisse

Ich zeige in der vorliegenden Studie, dass die Unterscheidung zwischen Unsicherheitsschocks und Sicherheitsschocks wichtig ist. Zum Beispiel verstärkt sich die Wirkung eines Unsicherheitsschocks, wenn man diesen von einem Sicherheitsschock abgrenzt. Darüber hinaus stelle ich fest, dass ein Unsicherheitsschock sowohl die Abwärtsrisiken als auch die Aufwärtsrisiken der Realwirtschaft vergrößert, wobei die Abwärtsrisiken deutlich stärker zunehmen.

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Abstract

I propose a Bayesian quantile VAR to identify and assess the impact of uncertainty and certainty shocks, unifying Bloom's (2009) two identification steps into one. I find that an uncertainty shock widens the conditional distribution of future real economic activity growth, in line with a risk shock. Conversely, a certainty shock (a shock strongly decreasing uncertainty) narrows the conditional distribution of future real activity growth. In addition to the difference in signs, I show that the two shocks are different shocks. Each shock impacts the real economy uniquely. I support this with the underlying events: For instance, uncertainty shocks relate to events such as Black Monday and 9/11, but also to fears about future negative economic outcomes. In contrast, certainty shocks often link to phases of irrational exuberance. Commonly, no distinction is made between uncertainty and certainty shocks. I show that uncertainty shocks become more important if distinguished from certainty shocks.

Keywords: Bayesian quantile VAR, uncertainty shocks, tail risks, irrational exuberance

JEL classification: C32, E44, G01.

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"Le doute n'est pas un état bien agréable, mais l'assurance est un état ridicule."
[Uncertainty is an uncomfortable position, but certainty is a ridiculous one.]
(Voltaire, 1785, p. 418)

1 Introduction

Major economic and political shocks, such as Black Monday or 9/11, dramatically increase uncertainty. Bloom (2009, hereafter Bloom) finds that these uncertainty shocks have a severe negative real impact. He identifies the dates of uncertainty shocks as right-hand tail realizations of stock market volatility (SMV).[1] Bloom reasons that the right-hand tail realizations are the truly exogenous shocks to uncertainty and provides a narrative for each shock identified (Black Monday, 9/11, etc.).

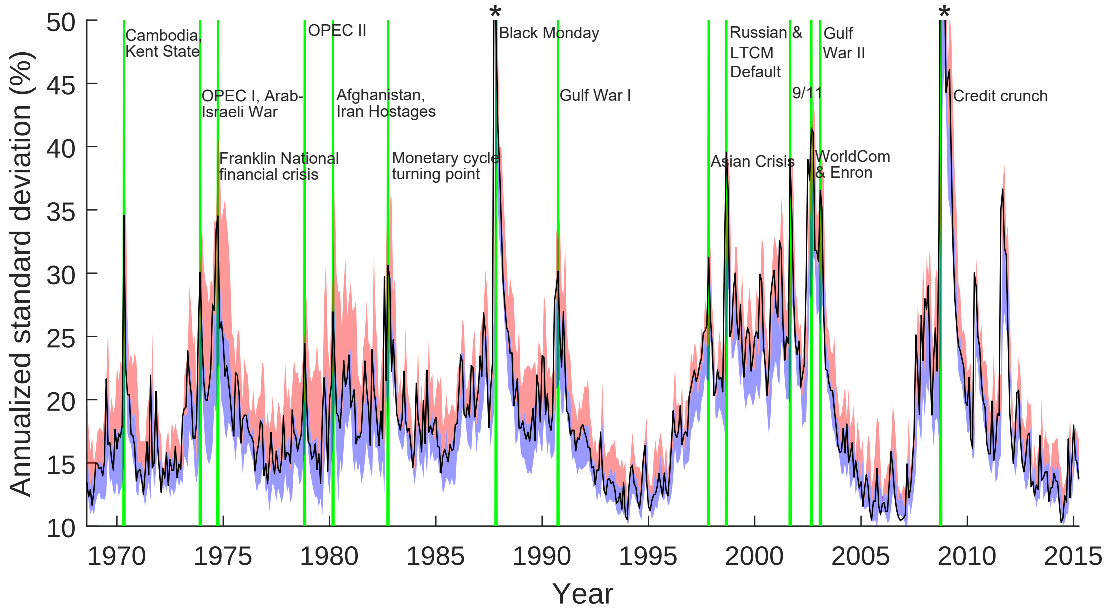


Figure 1: Monthly U.S. stock market volatility and its conditional distribution

Notes: The time series is Bloom's (2009) proxy of uncertainty (Chicago Board of Options Exchange VXO index from 1986 onwards; pre-1986 realized volatility of S&P 500 index). The upper edge of the red area is the time series' conditional right-hand tail (90% quantile), the lower edge of the blue area is its conditional left-hand tail (10% quantile), and the border of the red and blue area is its conditional mean. I describe the conditioning variables in Section 3.1. The green vertical lines refer to Bloom's (2009) dates of uncertainty shocks. The text to the right of each green line describes the underlying event as stated by Bloom (2009). For expositional purposes, the time series is capped at 50%. * indicates the times when this condition binds. Specifically, the time series exceeds this value around Black Monday and the credit crunch, reaching a maximum value of 58.2% and 64.4% respectively. LTCM is Long-Term Capital Management.

Figure 1 indicates a close link between Bloom's dates of uncertainty shocks (green vertical lines) and the conditional right-hand tail of SMV (90% quantile; upper edge of red area). Specifically, at the date of a shock, the right-hand tail realization of SMV (black line) tends to exceed its conditional right-hand tail. Therefore, I interpret uncertainty shocks as shocks to the conditional right-hand tail of SMV.

This interpretation extends our understanding of uncertainty shocks in at least two ways. First, Bloom identifies dates of uncertainty shocks as right-hand tail realizations of SMV, but he

[1] Bloom (2009) identifies these dates of uncertainty shocks as periods when SMV is more than 1.65 standard deviations above the Hodrick-Prescott detrended ($\lambda = 129,600$) mean of SMV.

does not asses their impact on the real economy through SMV.[2] He uses a two-step approach, essentially identifying uncertainty shocks from a dummy variable.[3] Therefore, Bloom possibly neglects an important transmission channel. Second, while many studies assess the impact of uncertainty shocks through SMV (or other proxies of uncertainty), they do not identify shocks to the conditional right-hand tail of SMV, but rather to the conditional expectation of SMV (border of red and blue area; Figure 1).[4] This could be restrictive because it implicitly assumes that shocks to the conditional right-hand tail and conditional left-hand tail (10% quantile; lower edge of blue area; Figure 1) have the same impact, but simply mirrored.

Against this backdrop, I ask three questions: First, is the conditional right-hand tail of SMV important for identifying and assessing the impact of uncertainty shocks? Second, is the impact of conditional left-hand tail shocks (certainty shocks) similar to the impact of conditional right-hand tail shocks (uncertainty shocks)? Considering the interpretation that uncertainty and certainty shocks are tail shocks, I ask: What is the impact of these two shocks on the conditional tails of macroeconomic variables? To shed light on these questions, I propose a Bayesian quantile vector autoregressive framework (BQVAR).

Three key results emerge. First, I find that uncertainty shocks widen the conditional distribution of real activity growth, which is in line with the interpretation that uncertainty shocks are risk shocks (see Christiano, Motto, and Rostango (2014)). Conversely, certainty shocks narrow the conditional distribution of real activity growth.

Second, I show that uncertainty and certainty shocks are different shocks, extending beyond the difference in signs. The conditional distributions of real economic variables respond uniquely to each shock. I support this via two external validation exercises. First, I illustrate that the two shocks distinctly correspond to an uncertainty-to-certainty ratio constructed from newspaper articles. Second, I find that my uncertainty shocks link to fundamental events (Black Monday, 9/11, etc.) as well as to fears about future fundamental events, such as economic slowdowns. In contrast, my certainty shocks often link to phases of irrational exuberance. For example, they relate to stock market records that have no link to any fundamental event.

Third, I find that the impact of uncertainty shocks is non-linear in two different ways. On the one hand, I show that uncertainty shocks are more important when identified to the conditional right-hand tail of SMV, rather than the conditional expectation of SMV or Bloom's dummy variable. Therefore, it is important to capture the transmissions channel through SMV and to distinguish uncertainty shocks from certainty shocks. On the other hand, I observe that uncertainty shocks impact the tails of real economic variables asymmetrically. For example, the conditional left-hand tail of real activity growth declines by more than double the amount the conditional right-hand tail increases after six months.

This paper relates to a vast literature discussing the impact of uncertainty shocks. Other studies also analyze the asymmetric effects of uncertainty, however, in different ways. For instance, Rossi and Sekhposyan (2015) derive a positive and negative uncertainty index by splitting forecast errors that undershoot or overshoot the conditional mean forecast. Segal, Shaliastovich, and Yaron (2015) construct a good and bad uncertainty index and show that the

[2] Note that Bloom (2009) evaluates the impact of uncertainty shocks identified through SMV in a robustness exercise. In doing so, he does not exclusively model the impact of his 17 dates of uncertainty shocks, i.e. the right-hand tail realizations.

[3] Bloom's (2009) dummy variable takes a value of one at the identified dates of uncertainty shocks and otherwise a value of zero. After identifying the dates of uncertainty shocks in a first step, he employs this dummy variable within a regular VAR framework to evaluate the real impact of uncertainty shocks in a second step.

[4] See, for instance, Bachmann, Elstner, and Sims (2013); Bekaert, Hoerova, and Lo Duca (2013); Colombo (2013); Jurado, Ludvigson, and Ng (2015); Baker, Bloom, and Davis (2016); Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016); Leduc and Liu (2016); Scotti (2016); Basu and Bundick (2017); Gorodnichenko and Ng (2017); Meinen and Roehe (2017); Carriero, Clark, and Marcellino (2018); Ludvigson, Ma, and Ng (2018); Hristov and Roth (2019).

former predicts an increase and the latter a decrease in future economic activity. Similar to Rossi and Sekhposyan (2015), the authors differentiate between positive and negative innovations to macroeconomic growth, therefore not distinguishing increases and decreases in second moments.

In a series of studies, Caggiano, Castelnuovo, and Groshenny (2014), Caggiano, Castelnuovo, and Nodari (2014), Caggiano, Castelnuovo, and Pellegrino (2017), and Caggiano, Castelnuovo, and Figueres (2017) employ non-linear VAR models and find that uncertainty shocks have different effects over the business cycle and at the zero lower bound. Furthermore, Allesandri and Mumtaz (2019), Popp and Zhang (2016), Mumtaz and Theodoridis (2018), and Mumtaz and Musso (2019) find that the impact of uncertainty shocks varies over the financial cycle, over geographic regions, and over time in general. While this research points out the time-dependent impact of uncertainty shocks, I distinguish the impact of strong increases in uncertainty versus strong reductions. I show that these are not specifically related to different phases of the business or financial cycle.

Finally, my BQVAR contributes to a nascent literature using single equation quantile regression to link financial conditions with the conditional tails of GDP growth (Adrian, Boyarchenko, and Giannone (2019)).[5] My multiple equation quantile regression framework may broaden our understanding of the complex macro-financial interdependencies. For instance, the identification and evaluation of structural shocks is vitally important for understanding the drivers of the conditional tails of the macroeconomic variables.[6]

The structure of the paper is as follows: Section 2 introduces the BQVAR framework. Section 3 discusses empirical issues, such as the data and the pseudo-structural analysis. Subsequently, I discuss the results in Section 4. In Section 5, I proceed with the external validation of uncertainty and certainty shocks. Section 6 concludes.

2 Bayesian quantile vector autoregression

Let $\mathbf{y}_t = (y_{1t}, y_{2t}, \ldots, y_{dt})'$ denote a vector of d endogenous random variables with $t = 1, \ldots, T$. For fixed quantile values $\boldsymbol{\tau} = (\tau_1, \tau_2, \ldots, \tau_d)'$, the reduced form quantile vector autoregressive framework (QVAR) can be written as

$$\mathbf{y}_t = \boldsymbol{\nu}_{\boldsymbol{\tau}} + \sum_{i=1}^{p} \mathbf{A}_{i|\boldsymbol{\tau}} \mathbf{y}_{t-i} + \mathbf{v}_{t|\boldsymbol{\tau}}, \tag{1}$$

where $\boldsymbol{\nu}_{\boldsymbol{\tau}}$ is a vector of intercepts, $\mathbf{A}_{i|\boldsymbol{\tau}}$ denotes the $d \times d$ matrix of lagged coefficients, and $\mathbf{v}_{t|\boldsymbol{\tau}} = (v_{1t|\tau_1}, v_{2t|\tau_2}, \ldots, v_{dt|\tau_d})'$ is a vector of error terms. The framework assumes that $Q_{\tau_j}(v_{jt|\tau_j}|\mathcal{F}_{t-1}) = 0$, where $j \in \{1, \ldots, d\}$. Q_{τ_j} refers to the τ_j-th quantile of $v_{jt|\tau_j}$ conditional on $\mathcal{F}_{t-1}$, which is the information set including information up to time period $t-1$.

To clarify notation, let $d = 2$ and $p = 1$. Given $\boldsymbol{\tau} = (\tau_1, \tau_2)'$, Equation (1) may be explicitly written as

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \nu_{1|\tau_1} \\ \nu_{2|\tau_2} \end{pmatrix} + \begin{pmatrix} a_{11,1|\tau_1} & a_{12,1|\tau_1} \\ a_{21,1|\tau_2} & a_{22,1|\tau_2} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} + \begin{pmatrix} v_{1t|\tau_1} \\ v_{2t|\tau_2} \end{pmatrix}. \tag{2}$$

[5]See also, for instance, Prasad, Elekdag, Jeasakul, Lafarguette, Alter, Xiaochen Feng, and Wang (2019); Adrian, Grinberg, Liang, and Malik (2018); Loria, Matthes, and Zhang (2019); Hartwig, Meinerding, and Schüler (2019).

[6]In this context, Chavleishvili and Manganelli (2019) provide an alternative to my BQVAR. The authors follow a frequentist approach and take a different path to identify structural shocks. Within my framework, one can identify shocks to different conditional quantiles of the endogenous variables. Furthermore, the Bayesian framework proposed in this paper allows the modelling of several variables. A more detailed comparison is provided in Section 2.5.

This emphasizes that the fixed quantile values may differ across the endogenous variables. For instance, τ_1 reflects the conditional quantile value of y_{1t}, i.e. $Q_{\tau_1}(y_{1t}|\mathcal{F}_{t-1}) = \nu_{1|\tau_1} + a_{11,1|\tau_1} y_{1,t-1} + a_{12,1|\tau_1} y_{2,t-1}$. This value can differ to the value of τ_2, the conditional quantile value of y_{2t}.

2.1 Pseudo-structural analysis

Within a regular VAR, one way to perform structural shock analysis is to orthogonalize the error terms. For instance, applying a Cholesky decomposition on the covariance matrix of error terms, yields uncorrelated structural shocks that allow for a structural interpretation.

Clearly, the covariance matrix is not of interest for structural shock analysis within the QVAR. This is because the covariance matrix summarizes the common fluctuation of error terms around means and not quantiles. In contrast, a co-exceedance measure in the spirit of Blomqvist (1950) and Koenker and Portnoy (1990) captures the common fluctuation of error terms around quantiles. It is written as

$$\boldsymbol{\Omega}_{\boldsymbol{\tau}} = (\omega_{jk}) \equiv \frac{\mathrm{E}[\psi_{\tau_j}(v_{jt|\tau_j})\psi_{\tau_k}(v_{kt|\tau_k})]}{f_{v_{jt|\tau_j}}(0) f_{v_{kt|\tau_k}}(0)}, \tag{3}$$

where $\psi_{\tau_j}(v_{jt|\tau_j}) \equiv \tau_j - \mathbb{1}(v_{jt|\tau_j} < 0)$, with $\mathbb{1}$ being the indicator function and $j, k \in \{1, \ldots, d\}$. Furthermore, $f_{v_{jt|\tau_j}}(0)$ denotes the pdf of $v_{jt|\tau_j}$ evaluated at 0. The product of $\psi_{\tau_j}(v_{jt|\tau_j})$ and $\psi_{\tau_k}(v_{kt|\tau_k})$ measures the strength with which the two error terms jointly exceed their respective quantiles contemporaneously.

Analogous to a regular VAR, Equation (3) allows for the identification of pseudo-structural disturbances $\boldsymbol{\varepsilon}_{t|\boldsymbol{\tau}} = (\varepsilon_{1t|\boldsymbol{\tau}}, \ldots, \varepsilon_{dt|\boldsymbol{\tau}})'$, where each $\varepsilon_{jt|\boldsymbol{\tau}}$ depends on the full vector of fixed quantiles $\boldsymbol{\tau}$. Specifically, I propose to identify structural shocks by the Cholesky decomposition $\boldsymbol{\Omega}_{\boldsymbol{\tau}} = P_{\boldsymbol{\tau}} P_{\boldsymbol{\tau}}'$ that yields

$$\boldsymbol{\varepsilon}_{t|\boldsymbol{\tau}} = P_{\boldsymbol{\tau}}^{-1} \widetilde{\psi}_{\boldsymbol{\tau}}(\mathbf{v}_{t|\boldsymbol{\tau}}), \tag{4}$$

where $\widetilde{\psi}_{\boldsymbol{\tau}}(\mathbf{v}_t) = (\psi_{\tau_1}(v_{1t|\tau_1})/f_{v_{1t|\tau_1}}(0), \ldots, \psi_{\tau_d}(v_{dt|\tau_d})/f_{v_{dt|\tau_d}}(0))'$. By construction, the pseudo-structural errors have mean zero and unit variance. Most importantly, they are uncorrelated with each other.

A pseudo quantile impulse response function (PQIRF) may then be defined in the spirit of Gallant, Rossi, and Tauchen (1993) and Koop, Pesaran, and Potter (1996):

$$\mathrm{PQIRF}_{\boldsymbol{\tau}}(h, \varepsilon_{jt|\boldsymbol{\tau}}, \mathcal{F}_{t-1}) = \breve{Q}_{\boldsymbol{\tau}}(\mathbf{y}_{t+h}|\varepsilon_{jt|\boldsymbol{\tau}}, \mathcal{F}_{t-1}) - Q_{\boldsymbol{\tau}}(\mathbf{y}_{t+h}|\mathcal{F}_{t-1}), \tag{5}$$

which allows to infer on the marginal impact of shock j on the system. That is, a baseline scenario $Q_{\boldsymbol{\tau}}(\mathbf{y}_{t+h}|\mathcal{F}_{t-1})$ is subtracted from a shock scenario $\breve{Q}_{\boldsymbol{\tau}}(\mathbf{y}_{t+h}|\varepsilon_{jt|\boldsymbol{\tau}}, \mathcal{F}_{t-1})$, where

$$\breve{Q}_{\boldsymbol{\tau}}(\mathbf{y}_t|\varepsilon_{jt|\boldsymbol{\tau}}, \mathcal{F}_{t-1}) \equiv \boldsymbol{\nu}_{\boldsymbol{\tau}} + \sum_{i=1}^{p} \mathbf{A}_{i|\boldsymbol{\tau}} \mathbf{y}_{t-i} + P_{\boldsymbol{\tau}} \begin{pmatrix} \vdots \\ \varepsilon_{jt|\boldsymbol{\tau}} \\ \vdots \end{pmatrix}. \tag{6}$$

For instance, assume $\varepsilon_{1t|\boldsymbol{\tau}} = \delta$ and all other shocks are zero, then

$$\mathrm{PQIRF}_{\boldsymbol{\tau}}(0, \varepsilon_{1t|\boldsymbol{\tau}} = \delta, \mathcal{F}_{t-1}) = P_{\boldsymbol{\tau}} \begin{pmatrix} \delta \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \tag{7}$$

$$\text{PQIRF}_{\boldsymbol{\tau}}(1, \varepsilon_{1t|\boldsymbol{\tau}} = \delta, \mathcal{F}_{t-1}) \equiv \mathbf{A}_{1|\boldsymbol{\tau}} P_{\boldsymbol{\tau}} \begin{pmatrix} \delta \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \text{ and so on.} \tag{8}$$

To illustrate, assume that $A_{i|\tau} = A_i$, where A_i is the i-th lagged dependent matrix of a regular VAR. This is the case if the conditional quantiles of the endogenous variables are just shifted in location relative to their conditional expectations. That is, the dynamic impact of the shock on the conditional expectations coincides with the dynamic impact of the shock on the conditional quantiles. Therefore, it is intuitive to see that PQIRF measures the impact of the shock on the (future) conditional quantiles of the endogenous variables.

In case $A_{i|\tau} \neq A_i$, the difference between this and regular impulse responses becomes apparent. As pointed out by Chavleishvili and Manganelli (2019), regular impulse responses measure the impact of a structural shock on the conditional expectation of the conditional expectations of future values of the endogenous variables. In contrast, PQIRF measures the impact of a structural shock on the conditional quantile of the conditional quantiles of future values of the endogenous variables. That is, PQIRF measures the deviation of conditional quantiles of the endogenous variables in response to a shock and how this deviation of the conditional quantiles leads to deviations of future conditional quantiles. This intertemporal dependence can be seen by considering the term $\breve{Q}_{\boldsymbol{\tau}}(\mathbf{y}_{t+1}|\varepsilon_{jt|\boldsymbol{\tau}}, \mathcal{F}_{t-1}) = \boldsymbol{\nu}_{\boldsymbol{\tau}} + \mathbf{A}_{1|\boldsymbol{\tau}} \breve{Q}_{\boldsymbol{\tau}}(\mathbf{y}_t|\varepsilon_{jt|\boldsymbol{\tau}}, \mathcal{F}_{t-1}) + \sum_{i=2}^{p} \mathbf{A}_{i|\boldsymbol{\tau}} \mathbf{y}_{t-i}$. It illustrates that conditional quantiles in $t+1$ depend on the conditional quantiles in t, and so on.

2.2 The multivariate Laplace distribution for multiple equation quantile regression

In this section, I introduce the multivariate Laplace distribution that I use for Bayesian estimation of the coefficient matrix $\mathbf{A}_{\boldsymbol{\tau}} = (\boldsymbol{\nu}_{\boldsymbol{\tau}}, \mathbf{A}_{1|\boldsymbol{\tau}}, \ldots, \mathbf{A}_{p|\boldsymbol{\tau}})'$, given fixed quantile values $\boldsymbol{\tau}$.

Proposition 1. Assuming that

$$\mathbf{v}_{t|\boldsymbol{\tau}} \sim \mathcal{L}_d(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}}, \mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}'), \tag{9}$$

where $\mathcal{L}_d$ denotes the general multivariate Laplace distribution, one can estimate the coefficient matrix $\mathbf{A}_{\boldsymbol{\tau}}$ for fixed quantile values $\boldsymbol{\tau}$. $\mathbf{m}_{\boldsymbol{\tau}}$ and the diagonal elements of $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ are

$$\mathbf{m}_{\boldsymbol{\tau}} = (m_j) = \frac{1 - 2\tau_j}{\tau_j(1 - \tau_j)} \quad \text{and} \quad \text{diag}(\boldsymbol{\Sigma}_{\boldsymbol{\tau}}) = (\sigma_{jj}^2) = \frac{2}{\tau_j(1 - \tau_j)}. \tag{10}$$

$\mathbf{B}$ is a positive definite matrix of size $(d \times d)$ defined as $\text{diag}(b_1, \ldots, b_d)$.

Proof. There exists a univariate Laplace distribution that is employed for single equation quantile regression (see e.g. Koenker and Machado (1999) or Yu and Moyeed (2001)). Since each component of a general multivariate Laplace admits a univariate representation (Kotz, Kozubowski, and Podgórski, 2001, Remark 6.3.2, p. 247), I generalize restrictions derived in the univariate case to the multivariate one.

To begin with, the univariate Laplace distribution employed for single equation quantile regression is:[7]

$$f_\tau(\eta_t) = \tau(1-\tau)\exp\{-\rho_\tau(\eta_t)\}, \text{ where } \rho_\tau(\eta_t) = \begin{cases} \eta_t \cdot \tau & \text{, if } \eta_t \geq 0 \\ \eta_t \cdot (\tau - 1) & \text{, if } \eta_t < 0. \end{cases} \tag{11}$$

Kotz et al. (2001) define the characteristic function of a general univariate Laplace as

$$\Psi_{\eta_t}(s) = \frac{1}{1 + \frac{1}{2}\sigma^2 s^2 - ims}, \tag{12}$$

where $m \in \mathbb{R}$, $\sigma \geq 0$, i is the imaginary unit, and s is an arbitrary real number. Therefore, the following restrictions on the parameters of the characteristic function can be derived:

$$\Psi_{\eta_t}(s) = E[\exp(is\eta_t)] \tag{13}$$

$$= \int_{-\infty}^{\infty} \exp(is\eta_t) f_\tau(\eta_t) d\eta_t \tag{14}$$

$$= \int_{-\infty}^{0} \tau(1-\tau)\exp(is\eta_t + (1-\tau)\eta_t)d\eta_t + \int_{0}^{\infty} \tau(1-\tau)\exp(is\eta_t - \tau\eta_t)d\eta_t \tag{15}$$

$$= \tau(1-\tau)\left(\frac{1}{is + (1-\tau)} + \frac{1}{\tau - is}\right) \tag{16}$$

$$= \frac{1}{1 + \frac{1}{\tau(1-\tau)}s^2 - i\frac{1-2\tau}{\tau(1-\tau)}s}, \tag{17}$$

or more specifically:

$$m = \frac{1-2\tau}{\tau(1-\tau)} \quad \text{and} \quad \sigma^2 = \frac{2}{\tau(1-\tau)}. \tag{18}$$

Next, the characteristic function of a general multivariate Laplace is

$$\Psi_{\boldsymbol{\eta}_t}(\mathbf{s}) = \frac{1}{1 + \frac{1}{2}\mathbf{s}'\boldsymbol{\Sigma}\mathbf{s} - i\mathbf{m}'\mathbf{s}}, \tag{19}$$

where $\boldsymbol{\eta}_t \in \mathbb{R}^d$, $\mathbf{m} \in \mathbb{R}^d$, $\boldsymbol{\Sigma}$ is a $(d \times d)$ nonnegative definite symmetric matrix, and $\mathbf{s}$ is a $(d \times 1)$ vector of arbitrary real numbers (see Kotz et al. (2001)). Therefore, the elements of $\mathbf{m}_{\boldsymbol{\tau}}$ and the diagonal elements of $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ have to fulfill the following restrictions:

$$m_j = \frac{1-2\tau_j}{\tau_j(1-\tau_j)} \quad \text{and} \quad \sigma_{jj}^2 = \frac{2}{\tau_j(1-\tau_j)}. \tag{20}$$

While the diagonal elements of $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ are restricted, the off-diagonal elements of $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ are not. These control the covariances between the univariate asymmetric Laplace distributions. The covariances can be decomposed into the product of the unrestricted correlations and the restricted variances, i.e. $\rho_{lk}\sigma_{\tau_l}\sigma_{\tau_k}$, where $l, k \in \{1, \ldots, d\}$ and $\sigma_{\tau_j} = \sqrt{\frac{2}{\tau_j(1-\tau_j)}}$. In this way, $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ may be

[7] This finding inspired research on quantile regression in the Bayesian context. For instance, Alhamzawi and Yu (2013) discuss conjugate priors and variable selection. Li, Xi, and Lin (2010) also discuss priors for the use of regularization, for example, lasso. Benoit and van den Poel (2012) offer a model for quantile regression in the case of a dichotomous response variable. Geraci and Bottai (2007), Liu and Bottai (2009), Geraci and Bottai (2014), Luo, Lian, and Tian (2012), Reich, Bondell, and Wang (2010), and Kobayashi and Kozumi (2012) present an approach for panel data. Chen, Gerlach, and Wei (2009) present an approach that accounts for heteroskedasticity that is, for example, present in financial data.

decomposed as

$$\boldsymbol{\Sigma}_{\boldsymbol{\tau}} = \mathbf{S}_{\boldsymbol{\tau}}\mathbf{R}\mathbf{S}_{\boldsymbol{\tau}}, \tag{21}$$

where $\mathbf{R}$ denotes the correlation matrix with ones on the diagonal and ρ_{lk} as off diagonal elements and $\mathbf{S}_{\boldsymbol{\tau}} = \text{diag}(\sigma_{\tau_1}, \ldots, \sigma_{\tau_d})$.

Besides the correlation structure in $\mathbf{R}$, the quantile restrictions imply a Laplace distribution with a variance that is completely defined through $\boldsymbol{\tau}$.[8] To relax this restriction, let $\mathbf{B}$ denote a scaling parameter that is defined by $\mathbf{B} = \text{diag}(b_1, \ldots, b_d)$. Following Kotz et al. (2001, p. 254), it holds that

$$\mathbf{v}_{t|\boldsymbol{\tau}} = \mathbf{B}\boldsymbol{\eta}_t \sim \mathcal{L}_d(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}}, \mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}'). \tag{22}$$

□

Proposition 2. Due to a mixture representation of the multivariate Laplace distribution (Kotz et al., 2001, p. 246), which is given by

$$\mathbf{v}_{t|\boldsymbol{\tau}} = \mathbf{B}\mathbf{m}_{\boldsymbol{\tau}} w_t + \sqrt{w_t}\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{1/2}\mathbf{z}_t, \tag{23}$$

one can use commonly known results for the estimation of $\mathbf{A}_{\boldsymbol{\tau}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ (see Section 2.3.1) as

$$\mathbf{y}_t | \mathbf{A}_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, w_t, \mathcal{F}_{t-1} \sim \mathcal{N}_d, \tag{24}$$

where w_t denotes a standard exponential random variable ($w_t \sim \mathcal{E}(1)$) and $\mathbf{z}_t$ a d-dimensional standard multivariate normal random variable ($\mathbf{z}_t \sim \mathcal{N}_d(\mathbf{0}, \mathbf{I}_d)$), with $\mathbf{I}_d$ being an identity matrix of dimension d. Additionally, let $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{1/2}$ represent the square root matrix $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ that yields $\left(\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{1/2}\right)\left(\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{1/2}\right)' = \boldsymbol{\Sigma}_{\boldsymbol{\tau}}$.

Proof. Equation (23) allows me to rewrite Equation (1) as

$$\mathbf{y}_t = \boldsymbol{\nu}_{\boldsymbol{\tau}} + \sum_{i=1}^{p} \mathbf{A}_{i|\boldsymbol{\tau}}\mathbf{y}_{t-i} + \mathbf{B}\mathbf{m}_{\boldsymbol{\tau}} w_t + \sqrt{w_t}\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{1/2}\mathbf{z}_t. \tag{25}$$

It follows that the conditional distribution of $\mathbf{y}_t$, given $\mathbf{A}_{\boldsymbol{\tau}}$, $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$, $\mathbf{B}$, w_t, and $\mathcal{F}_{t-1}$, is normal. The first two conditional moments of $\mathbf{y_t}$ are given by:

$$\text{E}[\mathbf{y}_t | \mathbf{A}_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, w_t, \mathcal{F}_{t-1}] = \boldsymbol{\nu}_{\boldsymbol{\tau}} + \sum_{i=1}^{p} \mathbf{A}_{i|\boldsymbol{\tau}}\mathbf{y}_{t-i} + \mathbf{B}\mathbf{m}_{\boldsymbol{\tau}} w_t = \boldsymbol{\mu}_{t|\boldsymbol{\tau}} \tag{26}$$

$$\text{V}[\mathbf{y}_t | \mathbf{A}_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, w_t, \mathcal{F}_{t-1}] = w_t\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}' = w_t\boldsymbol{\Sigma}_{\boldsymbol{\tau}\star}, \tag{27}$$

where $\boldsymbol{\Sigma}_{\boldsymbol{\tau}\star} = \mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}'$. Therefore, it holds that

$$\mathbf{y}_t | \mathbf{A}_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, w_t, \mathcal{F}_{t-1} \sim \mathcal{N}_d(\boldsymbol{\mu}_{t|\boldsymbol{\tau}}, w_t\boldsymbol{\Sigma}_{\boldsymbol{\tau}\star}). \tag{28}$$

□

2.3 Posteriors

This section introduces the conditional posterior distributions of $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$, $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}, w_t$, and $\mathbf{B}$, where $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$ denotes the column vector $\text{vec}(\mathbf{A}_{\boldsymbol{\tau}})$ of size $(d(dp+1) \times 1)$. To ease the exposition, I first cast

[8] The variance of the multivariate Laplace with quantile restrictions is given by $\mathbf{m}_{\boldsymbol{\tau}}\mathbf{m}_{\boldsymbol{\tau}}' + \boldsymbol{\Sigma}_{\boldsymbol{\tau}}$.

the VAR model in compact form:

$$\mathbf{y} = (\mathbf{I}_d \otimes \mathbf{X})\boldsymbol{\alpha}_{\boldsymbol{\tau}} + (\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}} \otimes \mathbf{I}_T)\mathbf{w} + \left(\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{1/2} \otimes \mathbf{W}^{1/2}\right)\mathbf{z}, \tag{29}$$

where $\mathbf{y} = \text{vec}(\mathbf{y}_1, \dots, \mathbf{y}_T)'$ is a $(Td \times 1)$ vector of observations, $\mathbf{X} = (\mathbf{x}_1', \dots, \mathbf{x}_T')'$ is a $(T \times (dp+1))$ matrix, where $\mathbf{x}_t = (1, \mathbf{y}_{t-1}', \dots, \mathbf{y}_{t-p}')$ represents a $(1 \times (dp+1))$ vector, $\mathbf{w} = (w_1, \dots, w_T)'$ is a $(T \times 1)$ vector and $\mathbf{W} = \text{diag}(\mathbf{w})$ reflects a $(T \times T)$ diagonal matrix. Therefore, $\mathbf{W}^{1/2} = \text{diag}(\sqrt{w}_1, \dots, \sqrt{w}_T)$. $\mathbf{z} = \text{vec}(\mathbf{z}_1, \dots, \mathbf{z}_T)$ denotes a $(Td \times 1)$ vector of multivariate standard normal random variables.

2.3.1 Conditional posteriors of $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$

I assume an independent normal-inverse-Wishart ($\mathcal{IW}$) prior:[9]

$$\boldsymbol{\alpha} \sim \mathcal{N}(\underline{\boldsymbol{\alpha}}, \underline{\mathbf{V}}) \quad \text{and} \quad \boldsymbol{\Sigma} \sim \mathcal{IW}(\underline{\boldsymbol{\Sigma}}, \underline{\nu}). \tag{30}$$

Prior times likelihood yields the standard posterior probability density functions:[10]

$$\boldsymbol{\alpha}_{\boldsymbol{\tau}} | \mathbf{y}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, \mathbf{w} \sim \mathcal{N}(\overline{\boldsymbol{\alpha}}_{\boldsymbol{\tau}}, \overline{\mathbf{V}}_{\boldsymbol{\tau}}) \quad \text{and} \quad \boldsymbol{\Sigma}_{\boldsymbol{\tau}} | \mathbf{y}, \boldsymbol{\alpha}_{\boldsymbol{\tau}}, \mathbf{B}, \mathbf{w} \sim \mathcal{IW}(\overline{\boldsymbol{\Sigma}}_{\boldsymbol{\tau}}, \overline{\nu}), \tag{31}$$

where

$$\overline{\mathbf{V}}_{\boldsymbol{\tau}} = [\underline{\mathbf{V}} + ((\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}')^{-\mathbf{1}} \otimes (\mathbf{X}'\mathbf{W}^{-\mathbf{1}}\mathbf{X}))]^{-1} \tag{32}$$

$$\overline{\boldsymbol{\alpha}}_{\boldsymbol{\tau}} = \overline{\mathbf{V}}_{\boldsymbol{\tau}}[\underline{\mathbf{V}}^{-\mathbf{1}}\underline{\boldsymbol{\alpha}} + ((\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}')^{-1} \otimes \mathbf{X}'\mathbf{W}^{-\mathbf{1}})(\mathbf{y} - (\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}} \otimes I_T)\mathbf{w})] \tag{33}$$

and

$$\overline{\nu} = \underline{\nu} + T \tag{34}$$

$$\overline{\boldsymbol{\Sigma}}_{\boldsymbol{\tau}} = \underline{\boldsymbol{\Sigma}} + (\mathbf{B}')^{-1}(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')'\mathbf{W}^{-1}(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')(\mathbf{B})^{-1}. \tag{35}$$

2.3.2 Conditional probability density function of the latent variable w_t

Proposition 3. The conditional probability density of w_t is proportional to

$$f(w_t | \mathbf{y_t}, \mathbf{A}_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, \mathcal{F}_{t-1}) \propto w_t^{-d/2} \exp\left(-\frac{1}{2}\left(a_{t|\boldsymbol{\tau}} w_t^{-1} + b_{\boldsymbol{\tau}} w_t\right)\right), \tag{36}$$

with $a_{t|\boldsymbol{\tau}} = (\mathbf{y}_t - \boldsymbol{\nu}_{\boldsymbol{\tau}} - \sum_{i=1}^p \mathbf{A}_{i|\boldsymbol{\tau}} \mathbf{y}_{t-i})'(\mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}')^{-1}(\mathbf{y}_t - \boldsymbol{\nu}_{\boldsymbol{\tau}} - \sum_{i=1}^p \mathbf{A}_{i|\boldsymbol{\tau}} \mathbf{y}_{t-i})$ and $b_{\boldsymbol{\tau}} = 2 + \mathbf{m}_{\boldsymbol{\tau}}' {\boldsymbol{\Sigma}_{\boldsymbol{\tau}}}^{-1} \mathbf{m}_{\boldsymbol{\tau}}$. This implies that w_t, conditional on the latter parameters, is proportional to a generalized inverse Gaussian with the following parameters:[11]

$$w_t | \mathbf{y}_t, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}, \mathbf{B}, \mathbf{A}_{\boldsymbol{\tau}}, \mathcal{F}_{t-1} \sim \mathcal{GIG}\left(-d/2 + 1, a_{t|\boldsymbol{\tau}}, b_{\boldsymbol{\tau}}\right). \tag{37}$$

Proof. Let $w \sim \mathcal{E}(1)$ and $\mathbf{y} \sim \mathcal{L}_d(\mathbf{m}, \boldsymbol{\Sigma})$. In order to show that the kernel of the $f(w|\mathbf{y})$ is

[9] I assume that all prior distributions are independent of the remaining parameters. For instance, for the prior of $\boldsymbol{\alpha}$ I assume that $f(\boldsymbol{\alpha}|\boldsymbol{\Sigma}, \mathbf{B}, w_t) = f(\boldsymbol{\alpha})$. As indicated, priors do not necessarily depend on the chosen quantiles $\boldsymbol{\tau}$.

[10] The decomposition of $(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')'\mathbf{W}^{-1}(\mathbf{Y} - \mathbf{X}\mathbf{A}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')$ into $(\mathbf{Y} - \mathbf{X}\hat{\mathbf{A}}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')'\mathbf{W}^{-1}(\mathbf{Y} - \mathbf{X}\hat{\mathbf{A}}_{\boldsymbol{\tau}} - \mathbf{w}(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}})')$ and $(\mathbf{A}_{\boldsymbol{\tau}} - \hat{\mathbf{A}}_{\boldsymbol{\tau}})\mathbf{X}'\mathbf{W}^{-\mathbf{1}}\mathbf{X}(\mathbf{A}_{\boldsymbol{\tau}} - \hat{\mathbf{A}}_{\boldsymbol{\tau}})$ also holds in this context.

[11] There are several algorithms available for the generation of random numbers from a generalized inverse Gaussian. I apply the one proposed by Devroye (2012) as it is computationally fast.

proportional to that of a generalized inverse Gaussian distribution, recall that the conditional density is obtained through

$$f(w|\mathbf{y}) = \frac{f(\mathbf{y}|w)f(w)}{f(\mathbf{y})}. \tag{38}$$

I have shown that $f(\mathbf{y}|w)$ has a multivariate normal pdf, i.e.

$$f(\mathbf{y}|w) = (2\pi)^{-d/2}|w\boldsymbol{\Sigma}|^{-1/2}\exp\left(-\frac{1}{2}(\mathbf{y}-\mathbf{m}w)'(w\boldsymbol{\Sigma})^{-1}(\mathbf{y}-\mathbf{m}w)\right). \tag{39}$$

Furthermore, $f(w) = \exp(-w)$. Neglecting $f(\mathbf{y})$ and the invariant terms of $f(\mathbf{y}|w)$,

$$\begin{aligned} f(w|\mathbf{y}) &\propto w^{-d/2}\exp\left(-\frac{1}{2}(\mathbf{y}-\mathbf{m}w)'(w\boldsymbol{\Sigma})^{-1}(\mathbf{y}-\mathbf{m}w) - w\right) && (40)\\ &= w^{-d/2}\exp\left(-\frac{1}{2}\left(\frac{\mathbf{y}'\boldsymbol{\Sigma}^{-1}\mathbf{y}}{w} - \mathbf{y}'\boldsymbol{\Sigma}^{-1}\mathbf{m} - \mathbf{m}'\boldsymbol{\Sigma}^{-1}\mathbf{y} + w\mathbf{m}'\boldsymbol{\Sigma}^{-1}\mathbf{m}\right) - w\right) && (41)\\ &\propto w^{-d/2}\exp\left(-\frac{1}{2}\left((\mathbf{y}'\boldsymbol{\Sigma}^{-1}\mathbf{y})w^{-1} + (2+\mathbf{m}'\boldsymbol{\Sigma}^{-1}\mathbf{m})w\right)\right). && (42) \end{aligned}$$

The probability density function of a generalized inverse Gaussian denoted by $\mathcal{GIG}(\lambda, \chi, \psi)$, with $\lambda = -(d/2)+1$, is given by

$$f(x|\lambda,\chi,\psi) = \frac{(\psi/\chi)^{\lambda/2}}{2K_\lambda(\sqrt{\chi\psi})}x^{\lambda-1}\exp\left\{-\frac{1}{2}(\chi x^{-1}+\psi x)\right\}, \tag{43}$$

where $K_\lambda(\cdot)$ reflects the modified Bessel function of the second kind. Therefore,

$$f(w|\mathbf{y}) \propto \mathcal{GIG}(-d/2+1, \mathbf{y}'\boldsymbol{\Sigma}^{-1}\mathbf{y}, 2+\mathbf{m}'\boldsymbol{\Sigma}^{-1}\mathbf{m}). \tag{44}$$

□

2.3.3 Conditional posterior of B

I assume a noninformative prior for $\mathbf{B}$, i.e. let

$$f(\mathbf{B}) = \text{constant}. \tag{45}$$

In this case, the conditional posterior of $\mathbf{B}$ follows the likelihood of a $\mathcal{L}_d(\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}\star})$, where $\boldsymbol{\Sigma}_{\boldsymbol{\tau}\star} = \mathbf{B}\boldsymbol{\Sigma}_{\boldsymbol{\tau}}\mathbf{B}'$. Following Kotz et al. (2001), it is:

$$\begin{aligned} f(\mathbf{B}|\mathbf{y},\boldsymbol{\alpha}_{\boldsymbol{\tau}},\boldsymbol{\Sigma}_{\boldsymbol{\tau}}) \propto \prod_{t=1}^{T} &\frac{2\exp\left((\mathbf{y}_t - \boldsymbol{A}'_{\boldsymbol{\tau}}\mathbf{x}'_t)'\boldsymbol{\Sigma}^{-1}_{\boldsymbol{\tau}\star}\mathbf{B}\mathbf{m}_{\boldsymbol{\tau}}\right)}{(2\pi)^{d/2}|\boldsymbol{\Sigma}_{\boldsymbol{\tau}\star}|^{1/2}}\left(\frac{(\mathbf{y}_t - \boldsymbol{A}'_{\boldsymbol{\tau}}\mathbf{x}'_t)'\boldsymbol{\Sigma}^{-1}_{\boldsymbol{\tau}\star}(\mathbf{y}_t - \boldsymbol{A}'_{\boldsymbol{\tau}}\mathbf{x}'_t)}{2+\mathbf{m}'_{\boldsymbol{\tau}}\boldsymbol{\Sigma}^{-1}_{\boldsymbol{\tau}}\mathbf{m}_{\boldsymbol{\tau}}}\right)^{(-d/2+1)} \\ &K_{(-d/2+1)}\left(\sqrt{(2+\mathbf{m}'_{\boldsymbol{\tau}}\boldsymbol{\Sigma}^{-1}_{\boldsymbol{\tau}}\mathbf{m}_{\boldsymbol{\tau}})((\mathbf{y}_t - \boldsymbol{A}'_{\boldsymbol{\tau}}\mathbf{x}'_t)'\boldsymbol{\Sigma}^{-1}_{\boldsymbol{\tau}\star}(\mathbf{y}_t - \boldsymbol{A}'_{\boldsymbol{\tau}}\mathbf{x}'_t))}\right), \end{aligned} \tag{46}$$

where $K_{(-d/2+1)}(\cdot)$ reflects the modified Bessel function of the second kind of order $-d/2+1$.

2.4 Metropolis-within-Gibbs sampler

The sampling of the $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$ coefficients and w_t is straightforward using a Gibbs sampler. The draws of the correlations contained in $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}$ and the scaling factors in $\mathbf{B}$ are more complex.

For the first case, I propose using the conditional posterior of $\mathbf{\Sigma}_{\boldsymbol{\tau}}$ and standardizing each draw.[12] Note that $\mathbf{\Sigma}_{\boldsymbol{\tau}}$ may be decomposed as

$$\mathbf{\Sigma}_{\boldsymbol{\tau}} = \mathbf{S}_{\boldsymbol{\tau}}\mathbf{R}\mathbf{S}_{\boldsymbol{\tau}}, \tag{47}$$

where $\mathbf{R}$ denotes the correlation matrix with ones on the diagonal and ρ_{lk} as off-diagonal elements and $\mathbf{S}_{\boldsymbol{\tau}} = \text{diag}(\sigma_{\tau_1}, \dots, \sigma_{\tau_d})$. Following this, each draw of $\mathbf{\Sigma}_{\boldsymbol{\tau}}$ can be rearranged as

$$\mathbf{R} = \mathbf{S}_{\boldsymbol{\tau}}^{-\mathbf{1}}\mathbf{\Sigma}_{\boldsymbol{\tau}}\mathbf{S}_{\boldsymbol{\tau}}^{-\mathbf{1}}. \tag{48}$$

What this achieves is that the diagonal elements of $\mathbf{\Sigma}_{\boldsymbol{\tau}}$ remain unchanged. This is important because quantile restrictions on the Laplace distribution have to remain fixed to obtain a consistent posterior for $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$. Having drawn the new correlation matrix $\mathbf{R}$, the covariance matrix $\mathbf{\Sigma}_{\boldsymbol{\tau}}$ can be updated using Equation (47).

In the case of $\mathbf{B}$, the posterior probability density function is rather complicated as the matrix appears in the mean and the variance of the conditional distribution of $\mathbf{y_t}$ (e.g. Equation (28)). Therefore, for the draw of $\mathbf{B}$, I propose to use a random walk Metropolis-Hastings (MH) algorithm. This simply requires that the conditional posterior probability density function can be evaluated (see Chib and Greenberg (1995)). In contrast to the Gibbs sampler, not every draw is accepted using the MH algorithm. At each draw, one calculates an acceptance probability and compares this to a random draw of a uniform random variable to decide on its acceptance. If not accepted, the previous draw is taken as the new draw. The acceptance probability is derived as in the following: Given a new draw of $\mathbf{B}$, called $\mathbf{B}^*$, and the last draw $\mathbf{B}^{(n-1)}$, where $n \in \{1, \dots, N\}$, it is

$$\alpha_{\text{MH},\mathbf{B}}(\mathbf{B}^{(n-1)}, \mathbf{B}^*) = \min\left[\frac{f\left(\mathbf{B}^*|\mathbf{y}, \boldsymbol{\alpha}_{\boldsymbol{\tau}}^{(n)}, \mathbf{\Sigma}_{\boldsymbol{\tau}}^{(n)}, \mathbf{w}^{(n)}\right)}{f\left(\mathbf{B}^{(n-1)}|\mathbf{y}, \boldsymbol{\alpha}_{\boldsymbol{\tau}}^{(n)}, \mathbf{\Sigma}_{\boldsymbol{\tau}}^{(n)}, \mathbf{w}^{(n)}\right)}, 1\right].^{13} \tag{49}$$

I calibrate the acceptance probability to be between 0.2 and 0.5.

In the following, I depict the algorithm for the case when draws of the scaling parameters are carried out jointly. Of course, this can be broken down into separate steps to ease the calibration of the acceptance rate. Furthermore, a random walk MH algorithm may be carried out using any symmetric distribution in the innovation part. This paper assumes a normal distribution.[14] Finally, I specify that the algorithm only accepts stationary draws of $\boldsymbol{\alpha}_{\boldsymbol{\tau}}$.

2.5 Relation to other approaches

The multivariate regression quantile model (VAR for VaR) proposed by White, Kim, and Manganelli (2015) nests the presented QVAR.[15] In a frequentist setting, White et al. (2015) provide the asymptotic theory for this class of models. Therefore, the presented framework differs by taking a Bayesian perspective.

[12] The other option would be to use a Metropolis-Hastings algorithm and sample the off-diagonal elements of $\mathbf{\Sigma}_{\boldsymbol{\tau}}$. A Gibbs sampler, however, is preferred as every draw is accepted: thus convergence is faster. I find that both options provide similar estimates.

[13] In the depiction of the acceptance probabilities, the draws of the other variables are also used as conditioning variables. Variables at draw (n) or $(n-1)$ are chosen in line with the algorithm presented in this section; however, they may of course vary according to the ordering in the sampler used.

[14] In practice, I draw the elements of $\mathbf{B}$ separately. For each draw, the scaling parameter c_d automatically adjusts in order to satisfy the acceptance ratio of 0.2 and 0.5 mentioned above.

[15] It represents a special case, as the presented QVAR model does not include lagged values of the conditional quantiles of the endogenous variables.

Algorithm 1 Metropolis-within-Gibbs sampler for Bayesian quantile VAR

A. Define prior distribution for $\boldsymbol{\alpha_\tau}$ and $\boldsymbol{\Sigma_\tau}$ and set starting values $\boldsymbol{\alpha}^0_{\boldsymbol{\tau}}, \boldsymbol{\Sigma}^0_{\boldsymbol{\tau}}$ and $\mathbf{B}^0$. Set variance of the random walk innovation used in the MH step, c.
B. Repeat for $n = 1, 2, \dots, N$.

1. Gibbs Step 1: For $t = 1, \dots, T$: Draw $w_t^{(n)} | \mathbf{y}_t, \boldsymbol{\alpha}_{\boldsymbol{\tau}}^{(n-1)}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n-1)}, \mathbf{B}^{(n-1)}$.
2. Gibbs Step 2: Draw $\boldsymbol{\alpha_\tau}^{(n)} | \mathbf{y}, \boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n-1)}, \mathbf{B}^{(n-1)}, \mathbf{w}^{(n)}$.
3. Gibbs Step 3: (i) Draw $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n)} | \mathbf{y}, \boldsymbol{\alpha}_{\boldsymbol{\tau}}^{(n)}, \mathbf{B}^{(n-1)}, \mathbf{w}^{(n)}$; (ii) Calculate $\mathbf{R}^{(n)} = \mathbf{S}_{\boldsymbol{\tau}}^{-1} \boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n)} \mathbf{S}_{\boldsymbol{\tau}}^{-1}$; (iii) Set $\boldsymbol{\Sigma}_{\boldsymbol{\tau}}^{(n)} = \mathbf{S}_{\boldsymbol{\tau}} \mathbf{R}^{(n)} \mathbf{S}_{\boldsymbol{\tau}}$.
4. MH Step 1: (i) Draw $\mathbf{v}_{**} \sim \mathcal{N}(\mathbf{0}, c\mathbf{I_d})$; (ii) Calculate $(b_1^*, \dots, b_d^*)' = \left(b_1^{(n-1)}, \dots, b_d^{(n-1)}\right)' + \mathbf{v}_{**}$; (iii) Evaluate $\alpha_{\mathrm{MH},\mathbf{B}}$; (iv) Draw $u_{**} \sim \mathcal{U}(0,1)$; (v) **If** $u_{**} \leq \alpha_{\mathrm{MH},\mathbf{B}}$ set $\left(b_1^{(n)}, \dots, b_d^{(n)}\right)' = (b_1^*, \dots, b_d^*)'$; (vi) **else** set $\left(b_1^{(n)}, \dots, b_d^{(n)}\right)' = \left(b_1^{(n-1)}, \dots, b_d^{(n-1)}\right)'$.

Additionally, White et al. (2015) derive PQIRF for one-off interventions to the endogenous variables. In this paper, I propose PQIRF for structural shocks, drawing on methods used for conventional structural VARs. Specifically, the narratives given to shocks through a Wold causal chain similarly apply in my framework.

In a recent paper, Chavleishvili and Manganelli (2019) propose a frequentist QVAR for forecasting and stress testing that builds on White et al. (2015). Their analysis focusses on two endogenous variables. In this paper, I show that modelling several variables is feasible within the Bayesian framework.

In parallel, Chavleishvili and Manganelli (2019) propose a different way of identifying structural shocks. Chavleishvili and Manganelli (2019) explicitly model the contemporaneous relation between the endogenous variables using a recursive structure to identify shocks. In such a framework, the fixed vector of quantiles should take the same value across equations in order to estimate the relevant contemporaneous relations between the endogenous variables. My approach is also suitable when the contemporaneous relation of shocks at different conditional quantiles of the endogenous variables is of interest. In parallel, I identify structural shocks by imposing restrictions on the contemporaneous relation of structural shocks and not the endogenous variables.

I compare the results of Chavleishvili and Manganelli's (2019) approach (see Appendix F) and mine in a robustness exercise, because the mapping between the two approaches is still subject to future research. In the context of my analysis, I find that the the two approaches yield very similar results.[16]

3 Empirical issues

In this section, I describe the data. After that, I present the setup of both the Bayesian estimation and the pseudo-structural analysis.

[16]Indeed, the impact on the conditional median of the real and policy variables is very similar using the recursive identification strategy. I do not report the results of the PFEVD, because structural errors are correlated using this approach.

3.1 Data

The monthly data set spans the period from 1968M04 to 2015M04.[17] First, I describe the different proxies of uncertainty used in this study. Second, I introduce the other variables that I use to investigate the impact of uncertainty and certainty shocks. Third, I discuss an uncertainty-to-certainty ratio that I construct to validate the identified uncertainty and certainty shocks. This ratio is not included in the BQVAR analysis.

Proxies of uncertainty: The main analysis of this paper uses Bloom's proxy of uncertainty (u_v or SMV). The proxy is the Chicago Board of Options Exchange VXO index, backcasted using the actual volatility of the S&P 500. It is backcasted because the VXO is not available prior to 1986.

Additionally, I consider Bloom's dummy variable (u_{Bloom}) that takes a value of one at Bloom's dates of uncertainty shocks and otherwise a value of zero. Therefore, it is based on u_v.[1] Bloom uses this dummy variable to identify and assess the impact of uncertainty shocks.

In a robustness exercise, I employ three further proxies of uncertainty (forecast dispersion, fundamental macroeconomic uncertainty, and fundamental financial uncertainty) to identify uncertainty shocks. Forecast dispersion is based on Bachmann et al. (2013) and measures expectations of future developments in general business activity. The other two proxies are argued to capture fundamental financial and macroeconomic uncertainty, focussing on the time-varying variance of the unforecastable component of a large set of variables. They have been suggested by Ludvigson et al. (2018) (LMN, u_f) and Jurado et al. (2015) (JLN, u_m). Appendix D contains more details on the variables and presents the results of the robustness exercise.

Other variables in the BQVAR: Alongside the proxies of uncertainty, I consider a subset of the variables used by Jurado et al. (2015) and Caldara et al. (2016).[18] Specifically, I measure economic activity by growth in real manufacturing industrial production (Δq), private consumption by growth in real personal consumption expenditure (PCE, Δc), inflation by growth in the PCE deflator (Δp), interest rates by percentage point changes in the effective federal funds rate (Δi), and equity markets by the return of the S&P 500 index (r).[19]

Uncertainty-to-certainty ratio for external validation of shocks: My uncertainty-to-certainty ratio captures whether there has been a change in public uncertainty narratives compared with certainty narratives from one month to another (see Figure 5 in Section 5). It is defined as the ratio of the number of newspaper articles with uncertainty narratives to the number of newspaper articles with certainty narratives and therefore has parallels to the indices proposed by Baker et al. (2016). It begins in 1985 because there is only a large set of newspapers available for the period after 1985.

The keywords for determining the number of articles with uncertainty narratives are: uncertainty, uncertain, fear, concern, panic, worry, doubt, and low. Mirroring these keywords, the keywords for determining the number of articles with certainty narratives are: certainty, certain,

[17] The sample is such that the different proxies of uncertainty used in this study cover the same time period. Specifically, the uncertainty measure proposed by Bachmann et al. (2013) restricts the earliest date of the sample and the indices of fundamental macroeconomic and financial uncertainty by Jurado et al. (2015) and Ludvigson et al. (2018) restricts the most recent. The choice of the sample period eliminates three of Bloom's (2009) dates of uncertainty shocks (Cuban missile crisis, assassination of JFK, and Vietnam buildup).

[18] Appendix A presents all variables, their transformations, and their time series plots.

[19] In a robustness exercise (not shown in the paper), I model the effective federal funds rate in levels. Results remain the same.

trust, faith, confidence, euphoria, hope, and high. Furthermore, I require that articles with uncertainty (certainty) narratives do not include any of the certainty (uncertainty) keywords. In addition to these keywords, I include keywords such that the index reflects newspaper articles that deal with both economic issues and stock markets. For more details on the construction of the ratio, see Appendix B.

Finally, the reason for including the keywords "uncertainty" and "uncertain" (and thus "certainty" and "certain") is because they are used in Baker et al. (2016). I determine the other keywords through my own reading of newspaper articles published in the month of a shock that is not preceded by another shock. I argue that this allows me to focus on the most relevant events, as consecutive shocks have a high chance of being related to the same event. Appendix C presents at least one representative newspaper headline for each of these months.

3.2 Estimation setup

Throughout the study I consider non-informative priors so that the data can determine the estimation of the parameters.[20] The priors are

$$\boldsymbol{\alpha} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{d(pd+1)} \cdot 10) \qquad \text{and} \qquad \boldsymbol{\Sigma} \sim \mathcal{IW}(d, \mathbf{I}_d).$$

I specify the model parsimoniously because I use non-informative prior information. Specifically, I choose a lag length of three.

Finally, the Metropolis-within-Gibbs sampler takes 15000 draws and discards the first 5000 draws as burn-in draws. I check trace plots and the quantile conditions on the error terms to assure the convergence of the sampler.

3.3 Pseudo-structural analysis

First, I describe the order of the Wold causal chain. Second, I present the assumptions on the conditional quantiles of the endogenous variables. The assumptions on the conditional quantiles can play an important role in the identification of structural shocks. I use them to distinguish uncertainty shocks from certainty shocks. Third, I briefly discuss the normalization of shocks and the pseudo forecast error variance decomposition (PFEVD).

Order of the Wold causal chain: The Cholesky decomposition of the co-exceedance measure given in Equation (3) implies a recursive structure of the shocks. I base the ordering of the Wold causal chain on the ordering in standard monetary policy VARs (Leeper, Sims, and Zha (1996); Christiano, Eichenbaum, and Evans (1999)). This entails arranging the real sector first (economic activity, consumption, prices), then specifying the policy sector (interest rate). I subsequently introduce the proxy of uncertainty and, lastly, I add the index of equity markets.

This order entails controlling for shocks to the real and policy sector when identifying uncertainty (certainty) shocks. Or alternatively, shocks to the real and policy sector may impact the uncertainty proxy contemporaneously. But uncertainty (certainty) shocks cannot impact the real and policy sector contemporaneously. Yet, uncertainty (certainty) shocks may impact equity markets contemporaneously.

Ordering the proxy of uncertainty after the real and policy sector is similar to the approach taken by Jurado et al. (2015). However, Jurado et al. (2015) do not assume that uncertainty (certainty) shocks may impact equity markets contemporaneously, but the other way around. Their assumption would be hard to justify for my setup. This is because my proxy of uncertainty

[20] In a robustness exercise, I increase the variance of the priors on the parameters of the model. The results remain unaffected, which supports the notion that the selected priors are indeed non-informative.

reflects the realized volatility of stock returns for some periods. Clearly, a shock to the realized volatility of stock returns almost certainly impacts stock returns over a given month. I assess whether this ordering justifies the interpretation of shocks as uncertainty (certainty) shocks using the PFEVD. It is justified if an uncertainty (certainty) shock mainly impacts SMV and not stock returns.

Assumptions on the conditional quantiles of the endogenous variables: I vary the assumptions on the conditional quantiles of SMV (τ_5) and stock returns (τ_6) to either identify uncertainty shocks or certainty shocks.

I identify uncertainty shocks by imposing $\tau_5 = 0.9$ and $\tau_6 = 0.1$. That is, an uncertainty shock reflects a shock that impacts the conditional right-hand tail of SMV and, moreover, may also impact the conditional left-hand tail of stock returns.

There are two arguments to this assumption: First, the uncertainty shocks identified by Bloom go in hand with strong declines on stock markets. Therefore, specifying τ_6 at the conditional median would underestimate the impact of uncertainty shocks, also relative to the conditional expectation. Medians downweigh tail-realizations relative to means. Second, Segal et al. (2015), Bekaert, Engstrom, and Ermolov (2015), and Bekaert and Engstrom (2017) report that there may be shocks that both increase volatility and improve economic conditions. An example is the increase in volatility during the technological boom in the U.S. prior to 2000. My identification scheme excludes these scenarios. I support these arguments through the results of my external validation exercise (see Section 5) and the results of my robustness exercise, which uses the conditional median of stock returns (see Appendix E).[21]

I identify certainty shocks assuming that the conditional quantiles of SMV and stock returns take the "opposing" quantile values relative to the identification of uncertainty shocks. Specifically, a certainty shock reflects a shock that impacts the conditional left-hand tail of SMV ($\tau_5 = 0.1$) and, moreover, may also impact the conditional right-hand tail of stock returns ($\tau_6 = 0.9$).

I evaluate the impact of uncertainty (certainty) shocks on the remaining variable considering three different parts of their conditional distributions. First, I benchmark the impact of uncertainty (certainty) shocks against the commonly employed approaches by analysing the conditional median of the remaining variables ($\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0.5$). Second, I study the impact of uncertainty (certainty) shocks on the conditional tails of the remaining variables. Specifically, I consider the conditional left-hand (right-hand) tail by assuming $\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0.1$ ($\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0.9$).

Further issues: I ensure the comparability of the PQIRF by normalising the size of an uncertainty (certainty) shock to one standard deviation of SMV. Furthermore, my analysis of the structural shocks in the BQVAR is similar to the analysis of structural shocks in Markov switching VARs. Specifically, I analyze the impact of an uncertainty (certainty) shock keeping the vector of quantile values fixed.

Finally, I also report a PFEVD based on the PQIRF. I construct the PFEVD using the unnormalized decomposition of the co-exceedance measure.

[21] Appendix E shows that the importance of an uncertainty shock decreases for stock returns and the real economy. In contrast, it increases for certainty shocks. Both results are in line with the arguments described above.

4 The impact of uncertainty and certainty shocks

First, I discuss the impact of uncertainty and certainty shocks on the conditional tails of stock market volatility and returns. Second, I describe their impact on the conditional distribution (median and tails) of the real and policy sector variables. Finally, I benchmark my results against the results of other approaches followed in the literature. For instance, I identify uncertainty shocks within a regular VAR using i) Bloom's dummy of uncertainty shocks and ii) SMV ("mean uncertainty shock").

Figure 2 and Figure 3 show the pseudo-impulse responses of an uncertainty shock and a certainty shock until twelve months after the shock. Figure 4 plots the corresponding impulse responses of a Bloom uncertainty shock and a mean uncertainty shock. Table 9 sets forth the corresponding (pseudo) forecast error variance decompositions, showing the average variance explained from zero to six months and seven to twelve months after the shock. Furthermore, I compare my results to uncertainty shocks identified by other proxies of uncertainty. Appendix D contains the corresponding graphs and tables.

4.1 The impact on stock market volatility and returns

An uncertainty shock has a different impact on SMV and stock returns than a certainty shock (e.g. Figure 2). An uncertainty shocks leads to a persistent rise of the conditional right-hand tail of SMV and a persistent decline of the conditional left-hand tail of stock returns.

In contrast, the impact of a certainty shock is much less persistent. In parallel, the sign of the impact on stock returns changes over the horizon. Taken together, this means that a certainty shock induces a rather temporary decline of the left-hand tail of SMV and in tandem leads to a damped oscillation of the right-hand tail of stock returns. In the first month, the right-hand tail of stock returns increases by about 1.6 percentage points (p.p.), but declines thereafter.

The PFEVD suggests that the identification scheme successfully recovers shocks that are most relevant for SMV (Table 9). Both shocks explain most of the variation in SMV. For the first half-year, the average ranges from 86.6% to 93.6%. In parallel, both shocks are less important for fluctuations in stock returns. Yet, an uncertainty shock is much more important for the left-hand tail of stock returns than a certainty shock is for the right-hand tail of stock returns (35.3-41.4% vs. 1.9-2.9%).

4.2 The impact on the conditional median of real and policy sector variables

An uncertainty shock has a different impact on the conditional median of the real economy than a certainty shock (Figure 2). For instance, an uncertainty shock accounts for a persistent and strong decline of the conditional median of real activity growth. It accounts for about 6.7% of fluctuations of real activity growth within the first six months. The cumulated fall in real activity is about -1% until month six.

In contrast, a certainty shock leads to an only modest increase of the conditional median of real activity growth over six months. Thereafter, the conditional median of real activity growth decreases, destroying the previous gains almost completely. Cumulating only the significant responses, the gain in activity amounts to only 0.08% after twelve months. This is also reflected in the PFEVD: A certainty shock only accounts for 0.2% of fluctuations in the conditional median of real activity growth.

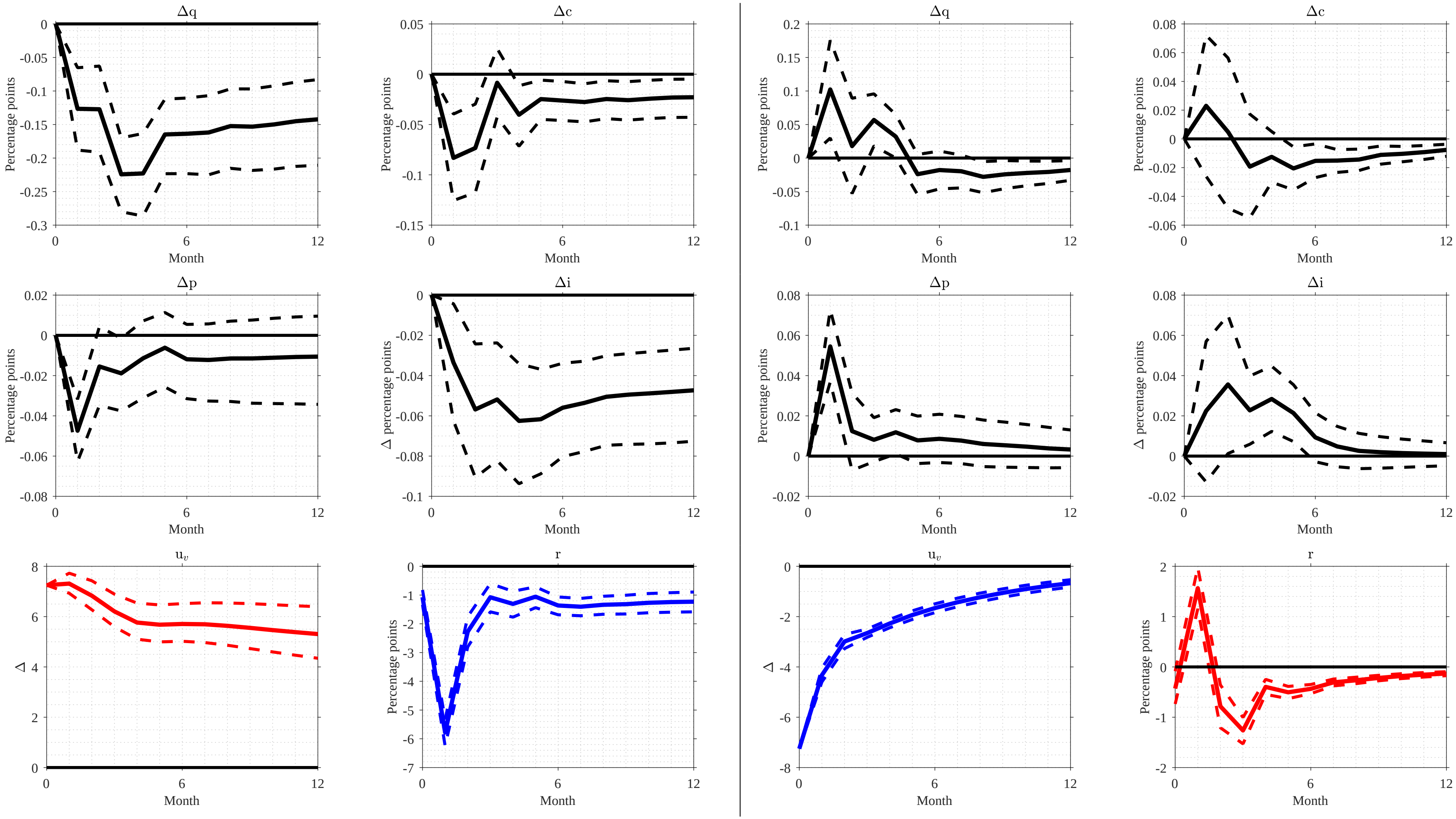


Figure 2: The impact of an uncertainty shock (left panel) and a certainty shock (right panel) on the conditional median of real and policy sector variables

Notes: The panels depict the pseudo-impulse responses of the conditional median of the macroeconomic and policy sector variables (Δq, Δc, Δp, Δi) and the conditional tails of the financial sector variables (u_v, r) in response to an uncertainty and a certainty shock. Specifically, I identify an uncertainty shock to the conditional 0.9 and 0.1 quantiles of u_v and r respectively. I identify a certainty shock to the conditional 0.1 and 0.9 quantiles of u_v and r respectively. The color blue (red) marks the conditional 0.1 (0.9) quantile. For more details on shock identification, see Section 3.3. Solid lines refer to the median of the distribution of the impulse responses and dashed lines correspond to posterior 68% probability bands. Δq denotes growth in industrial production, Δc is growth in consumption, Δp is inflation, Δi is changes in the interest rate, u_v is the proxy of uncertainty (stock market volatility), and r denotes stock returns.

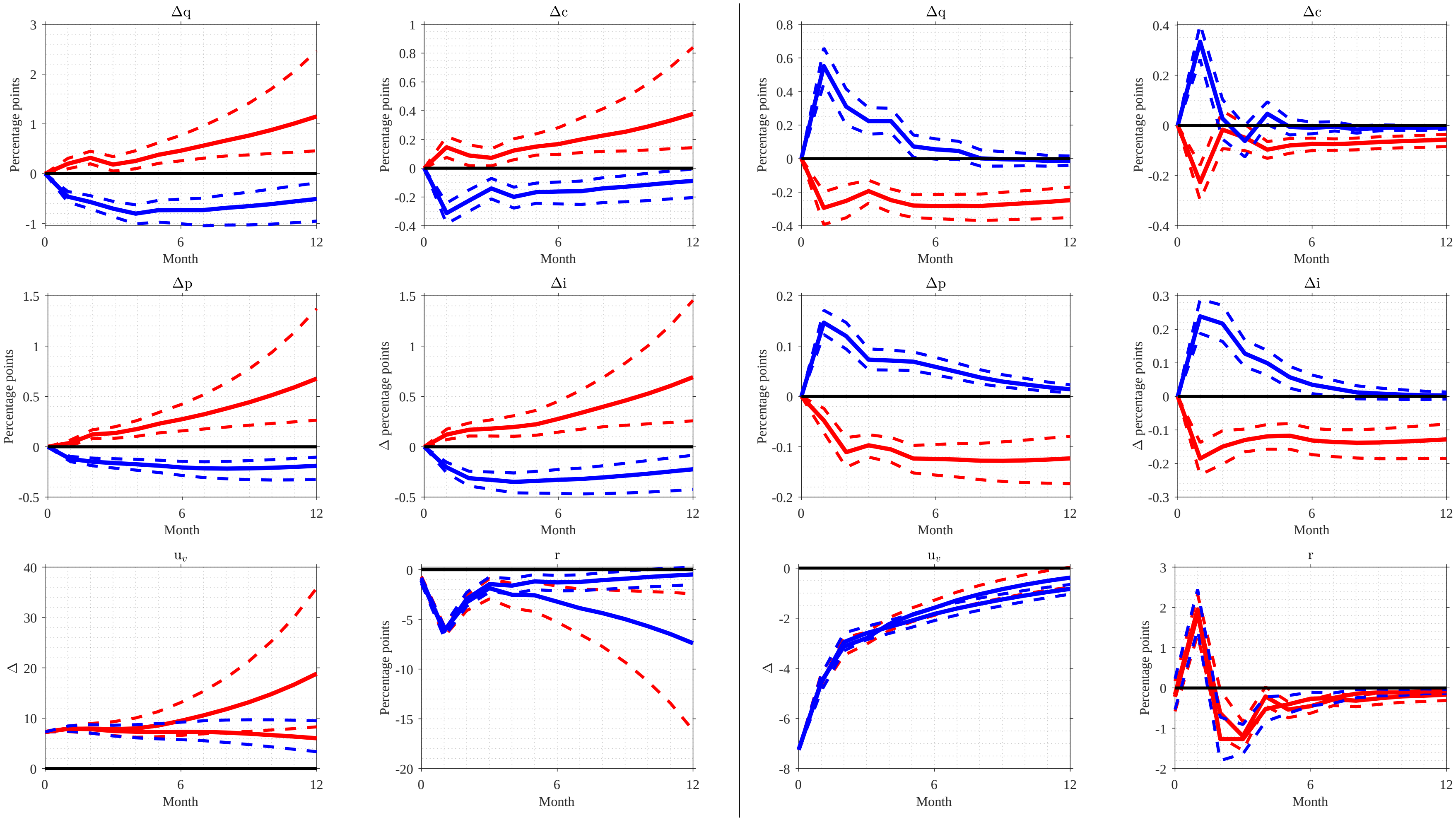


Figure 3: The impact of an uncertainty shock (left panel) and a certainty shock (right panel) on the conditional (left-hand and right-hand) tails of real and policy sector variables

Notes: The panels depict the pseudo-impulse responses of the conditional 0.1 quantile and the conditional 0.9 quantile of the real and policy sector variables (Δq, Δc, Δp, Δi) and the conditional tails of the financial sector variables (u_v, r) in response to an uncertainty and a certainty shock. Specifically, I identify an uncertainty shock to the conditional 0.9 and 0.1 quantiles of u_v and r respectively. I identify certainty shocks to the conditional 0.1 and 0.9 quantiles of u_v and r respectively. The color blue (red) marks the conditional 0.1 (0.9) quantile. For more details on shock identification, see Section 3.3. Solid lines refer to the median of the distribution of the impulse responses and dashed lines correspond to posterior 68% probability bands. In the left panel, responses of u_v and r with blue (red) dashed lines represent the responses when considering the 0.1 (0.9) conditional quantile of macroeconomic variables. In the right panel, responses of u_v and r with red (blue) dashed lines represent the responses when considering the 0.9 (0.1) conditional quantile of macroeconomic variables. Δq denotes growth in industrial production, Δc is growth in consumption, Δp is inflation, Δi is changes in the interest rate, u_v is the proxy of uncertainty (stock market volatility), and r denotes stock returns.

Table 1: Variance explained by various shocks (in percent)

Shock	Uncertainty			Certainty			Bloom	Mean
Quantile	0.5	0.1	0.9	0.5	0.1	0.9	-	-
Average 0-6 months								
Δq	6.7	23.2	9.4	0.2	2.7	1.9	2.8	3.8
Δc	2.1	13.7	5.2	0.0	2.3	1.0	0.4	1.0
Δp	2.5	23.4	11.6	0.5	4.4	1.5	0.4	1.8
Δi	7.6	19.7	10.3	0.4	2.9	2.1	2.8	0.6
u_v	93.6	89.5	86.8	92.0	88.3	90.1	94.2	96.2
r	35.3	39.7	41.4	1.9	2.9	2.2	8.1	20.8
Average 7-12 months								
Δq	17.1	36.8	32.3	0.3	3.2	4.6	3.7	7.4
Δc	3.4	20.8	22.0	0.1	2.7	1.7	0.6	1.4
Δp	3.1	44.3	34.6	0.5	6.8	4.1	0.8	1.8
Δi	21.1	34.7	31.8	0.6	3.8	4.0	3.9	1.4
u_v	79.7	70.1	62.4	83.7	79.2	77.2	93.1	94.7
r	40.6	41.9	44.8	2.8	3.8	3.1	8.4	23.3

Notes: The table shows the pseudo forecast error variance decompositions of an uncertainty and a certainty shock for different parts of the conditional distribution (median (0.5 quantile), left-hand tail (0.1 quantile), and right-hand tail (0.9 quantile)) of the real and policy sector variables (Δq, Δc, Δp, Δi) and the conditional tails of the financial sector variables (u_v, r). Specifically, I identify an uncertainty shock to the conditional 0.9 and 0.1 quantiles of u_v and r respectively. I identify a certainty shock to the conditional 0.1 and 0.9 quantiles of u_v and r respectively. The color blue (red) marks the conditional 0.1 (0.9) quantile. For more details on the shock identification, see Section 3.3. In parallel, the table shows the forecast error variance decomposition (FEVD) of a Bloom (2009) uncertainty shock, identified on the basis of Bloom's (2009) dummy variable within a regular VAR. Furthermore, the table shows the FEVD of a mean uncertainty shock, identified through u_v within a regular VAR model. Δq denotes growth in industrial production, Δc is growth in consumption, Δp is inflation, Δi is changes in the interest rate, u_v is the proxy of uncertainty (stock market volatility), and r denotes stock returns.

The monetary authority reacts much more strongly and more immediately to an uncertainty shock than to a certainty shock. The conditional median of interest rate changes declines in response to an uncertainty shock and rises in response to a certainty shock. An uncertainty shock accounts for about 7.6% of the variance in interest rate changes. The corresponding figure for a certainty shock only amounts to 0.4%.

4.3 The impact on the conditional tails of real and policy sector variables

In line with the interpretation that an uncertainty shock is a risk shock, an uncertainty shock persistently widens the conditional distributions of the real and policy sector variables (Figure 3). In fact, the widening is asymmetric. For example, the conditional left-hand tail of real activity growth decreases by more than double the increase of its conditional right-hand tail in absolute values and six months after the shock.

In contrast, a certainty shock narrows the conditional distribution of the real and policy sector variables, but in a different way to an uncertainty shock. The impact differs in that a certainty shock only temporarily lifts the left-hand tail, for instance, for real activity growth. The right-hand tail is persistently lowered.

The PFEVD indicates that both shocks are more important for the conditional tails of the real and policy sector variables than for the medians. In line with their asymmetric impact, both shocks are more important for the conditional left-hand tails than for the right-hand tails. For instance, an uncertainty shock accounts for 23.2% (9.4%) of variation of the conditional left-hand (right-hand) tail of real activity growth over the first half-year.

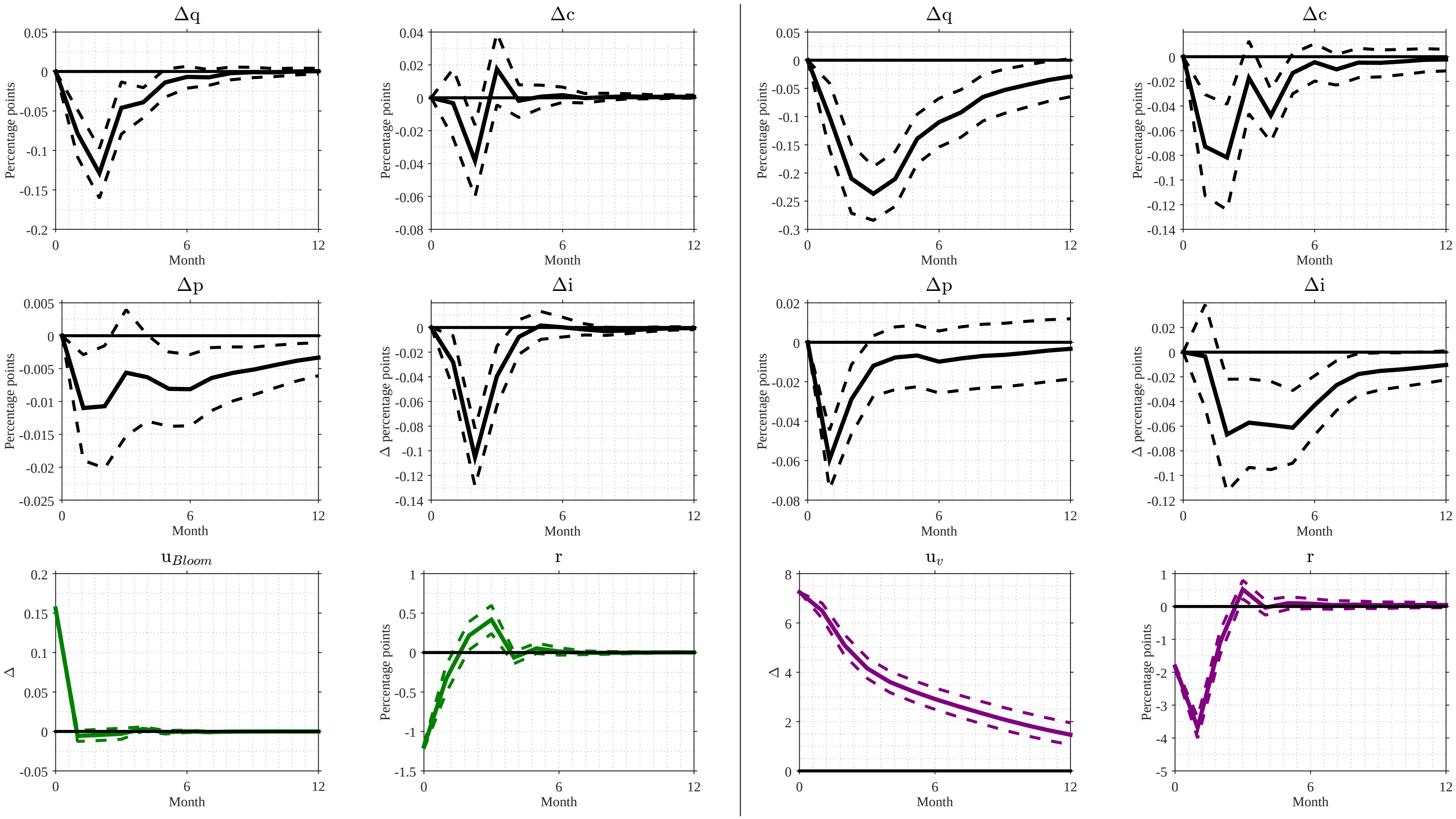


Figure 4: The impact of a Bloom uncertainty shock (left panel) and a mean uncertainty shock (right panel)

Notes: The panels depict regular impulse responses of the real and policy sector variables (Δq, Δc, Δp, Δi) and the financial sector variables (u_v, r) in response to a Bloom (2009) uncertainty shock and a mean uncertainty shock, both identified within a regular VAR. A Bloom (2009) uncertainty shock is a shock identified using Bloom's (2009) dummy variable. The dummy variable takes a value of one at Bloom's (2009) dates of uncertainty shocks and zero otherwise. A mean uncertainty shock is a shock identified to the conditional expectation of u_v. Solid lines refer to the median of the distribution of the impulse responses and dashed lines correspond to posterior 68% probability bands. Δq denotes growth in industrial production, Δc is growth in consumption, Δp is inflation, Δi is changes in the interest rate, u_v is the proxy of uncertainty, and r denotes stock returns.

4.4 A comparison with other uncertainty shocks

First, I compare my results with the impact of an uncertainty shock identified with Bloom's dummy variable within a regular VAR. Second, I contrast my findings with the impact of an uncertainty shock identified using SMV within a regular VAR. Third, I relate my findings to the the impact of uncertainty shocks when using other proxies of uncertainty (forecast dispersion, fundamental macroeconomic uncertainty, and fundamental financial uncertainty).

A Bloom uncertainty shock has a much weaker impact than my uncertainty shock (Figure 4). This is because Bloom's dummy variable does not capture the transmission through SMV. Specifically, a Bloom shock leads to a one-off effect on the dummy of uncertainty, instead of a persistent rise of the conditional right-hand tail of SMV. Therefore, the Bloom shock only induces a brief decline of stock returns followed by a short rise. The shock explains only 8.1% of the variation in stock returns over the first half-year; relative to 35.3% (uncertainty shock on conditional median; Table 9). Consequently, the impact on the real economy is much weaker than the impact of my uncertainty shock. For instance, the impact on real activity becomes insignificant as early as in the fifth month after the shock. It explains only 2.8% of variation over the first half-year, compared with 6.7%. The impact on interest rate changes is also temporary.

A mean uncertainty shock has a weaker impact than my uncertainty shock as well. The impact of a mean uncertainty shock ranges midway between the impact of an uncertainty shock and a certainty shock (Figure 4). One of the reasons for this is that the persistence of the SMV response lies between the persistence of the left-hand and right-hand tail responses. This suggests that a mean uncertainty shock combines uncertainty and certainty shocks, which is a reason why a mean uncertainty shock leads to a weaker impact than my uncertainty shock. This is supported by the (P)FEVD (Table 9). For instance, a mean uncertainty shocks explains 3.8% of variation of real activity growth over the first half-year, which is about halfway between 6.7% and 0.2% (variation explained by uncertainty and certainty shock).

The other proxies analyzed in this study (see Appendix D) suggest two further findings: i) Identified through forecast dispersion, uncertainty shocks are not more important when identified to the conditional right-hand tail. These shocks appear to be linear; ii) the impact of uncertainty shocks identified through fundamental macroeconomic and fundamental financial uncertainty is similar to the impact of a mean uncertainty shock. This seems intuitive as both measures are likely to combine uncertainty and certainty shocks.

5 An external validation of uncertainty and certainty shocks

The previous analysis suggests that uncertainty and certainty shocks are different shocks. This section aims to externally validate this finding.

First, I report that the typical event that underlies the two shocks differs (see Appendix C for the collection of newspaper headlines). Second, I show that an externally constructed uncertainty-to-certainty ratio corresponds distinctly with each of the shocks (Figures 5 and 6). Third, I provide evidence that both shocks are reflected in the measures of fundamental macroeconomic (u_m) and financial (u_f) uncertainty (Figure 7).

In the following, I refer to the right-hand (left-hand) tail exceedances when discussing uncertainty (certainty) shocks (red/blue spikes in Figure 5). Exceedances are of interest as they mark the strongest 10% of shocks, providing information about the location of the conditional right-hand (left-hand) tail and, thus, determining the dynamic responses of the system.

My uncertainty shock occurs on most, but not all, of Bloom's dates of uncertainty shocks (see Figure 5). Specifically, three of Bloom's dates are not present in my analysis, suggesting that they

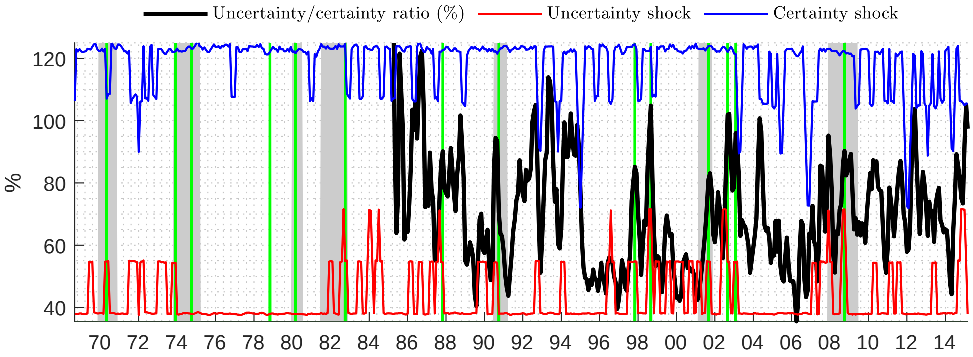


Figure 5: Uncertainty-certainty ratio and the uncertainty / certainty shock series

Notes: The uncertainty-to-certainty ratio indicates the number of newspaper articles with uncertainty narratives relative to the number of articles with certainty narratives during a specific month. For more details on the ratio, see Section 5 and Appendix B. The uncertainty-to-certainty ratio and the uncertainty/certainty shock series are smoothed using a three-month moving average. The transformation is applied in order to reduce noise that is present in the two series. Green lines reflect Bloom's (2009) dates of uncertainty shocks and the grey areas reflect NBER recession periods.

are explained by the real or policy sector.[22] The other months of uncertainty shocks typically relate to economic or political events such as the downgrading of the U.S. credit rating (August 2011) or the political future of President Richard Nixon (November 1973). Environmental events also increase uncertainty, such as the nuclear crisis in Japan (March 2011). In parallel, I find that fears regarding future negative outcomes increase uncertainty. For instance, in October 1989, fears about future negative real outcomes increase uncertainty sending stock markets tumbling similar to Black Monday. This emphasizes the forward-looking nature of financial markets.

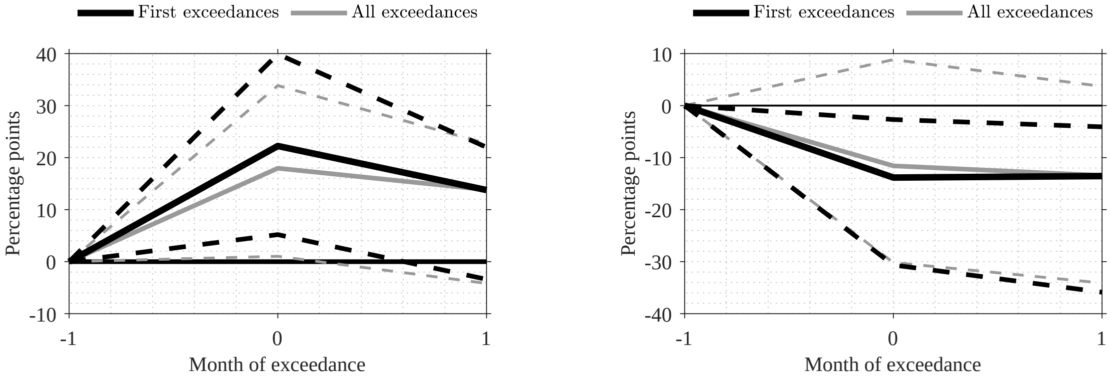
My certainty shock typically refers to the absence of a (negative) event that occurs together with strong increases or even records on stock markets. Clearly, records do not reflect fundamental news. Therefore, I conclude that certainty shocks relate to periods of irrational exuberance. For instance, the Dow hitting 1000 for the first time in history (November 1972) concerns a month of a certainty shock. Nevertheless, some of the shocks also reflect fundamental news. For instance, a certainty shock can be observed in connection with peace hopes for Vietnam (August 1968) and a decline in oil prices (June 2005).

The uncertainty-to-certainty ratio increases with uncertainty shocks and decreases with certainty shocks (Figure 6). That is to say, there is a relative increase in uncertainty (certainty) narratives relative to certainty (uncertainty) narratives during months of uncertainty (certainty) shocks. The increase (decrease) is stronger for months with first exceedances, i.e. months where shocks are not preceded by other shocks in the month before.[23] Finally, Figure 5 suggests shocks do not necessarily relate to business cycle fluctuations, as they occur over the entire sample and not only during NBER recession periods (grey area).

Finally, both the uncertainty and certainty shocks are reflected in the two proxies of uncertainty u_m and u_f (Figure 7). Specifically, there are periods where increases (decreases) correspond to dates of uncertainty (certainty) shocks. For instance, the strong decreases of u_f

[22] The events that are missing are the Franklin National financial crisis shock (October 1974), the OPEC II oil price shock (November 1978), and the Afghanistan war / Iran hostages shock (March 1980).

[23] Within the category *first exceedances*, 47 out of 56 are uncertainty shocks and 32 out of 56 instances are certainty shocks. The category *first exceedances* allows me to focus on the arguably most relevant events, as consecutive exceedances have a high chance of being related to the same event. Note that the number 56 is the outcome of considering the exceedances of the conditional 90% (10%) quantile. One can reduce this number by considering higher (lower) quantile values. However, a smaller number also means that there is more uncertainty when estimating tail dynamics.



(a) Uncertainty shock

(b) Certainty shock

Figure 6: Uncertainty-to-certainty ratio development around dates of exceedances

Notes: Exceedances are the red/blue spikes in Figure 5 and refer to times when a structural shock exceeds its conditional quantile. First exceedances define exceedances that are not preceded by another exceedance (left-hand tail or right-hand tail) in the month before. -1 (1) indicates the month prior to (past) the exceedance. The solid line is the median development of the ratio around dates of exeedances. Dashed lines mark the 75% and 25% quantiles.

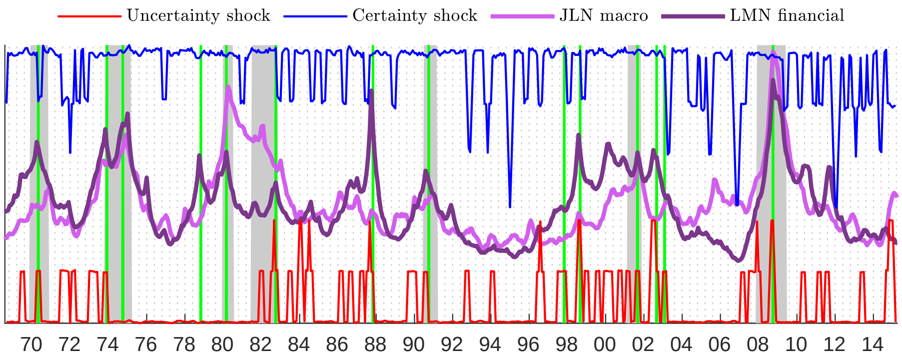


Figure 7: The uncertainty/certainty shock series in relation to measures of uncertainty

Notes: JLN macro is the measure of fundamental macroeconomic uncertainty introduced by Jurado et al. (2015). LMN financial is the measure of fundamental financial uncertainty proposed by Ludvigson et al. (2018). The uncertainty and certainty shock series are smoothed using a three-month moving average. The transformation is applied in order to reduce noise that is present in the two series. Green lines reflect Bloom's (2009) dates of uncertainty shocks and the grey areas NBER recession periods.

around 2012 and prior to the global financial crisis coincide with a certainty shock. This is in line with the evidence that a mean uncertainty shock combines uncertainty and certainty shocks (see Section 4.4).

6 Concluding remarks

In this study, I propose a Bayesian quantile VAR framework (BQVAR) for identifying and assessing the impact of uncertainty and certainty shocks.

I find that an uncertainty shock widens the future conditional distribution of real economic variables, which is in line with the interpretation that an uncertainty shock is a risk shock. On the contrary, a certainty shock narrows the conditional distribution of real economic variables.

In addition to the difference in signs, I show that an uncertainty shock and a certainty shock are two different shocks, for instance, in terms of their impact on the real economy. I support this through two external validation exercises. The first is based on textual analysis of newspaper articles. For the second, I report events associated with each of the two shocks. While the events associated with uncertainty shocks reflect common dates of uncertainty shocks, such as Black Monday, the events linked to certainty shocks do not. Certainty shocks are rather associated with phases of irrational exuberance.

Finally, I find that the impact of an uncertainty shock is non-linear in two different ways. On the one hand, I show that an uncertainty shock is more important when identified to the conditional right-hand tail of SMV instead of the conditional expectation of SMV or Bloom's dummy variable. Therefore, it is important to capture the transmissions channel through SMV and to distinguish uncertainty shocks from certainty shocks. On the other hand, I observe that an uncertainty shock impacts the tails of real economic variables asymmetrically.

Three policy conclusions follow. First, policy makers and researchers should take into account the non-linear impact of uncertainty shocks on the conditional median as well as on the conditional tails of macroeconomic variables. For instance, this may be crucial when calibrating models that inform the role of monetary policy in offsetting spikes in uncertainty. Second, the BQVAR appears to be a promising framework for the analysis of the complex and non-linear macro-financial linkages. For instance, I show how uncertainty shocks move the conditional tails of macroeconomic variables differently. Such analysis may help policy makers to better understand the trade-offs faced when taking decisions on the use of instruments. Third, certainty shocks that link to phases of irrational exuberance underscore the importance of policy actions in response to such shocks.

I conclude this paper by drawing a link to the field of social psychology. Findings in this area may provide an explanation for the differing events related to uncertainty and certainty shocks (see Tiedens and Linton (2001) and references therein). Specifically, it is argued that uncertainty encourages systematic information processing, implying agents that only pay attention to fundamental shocks. In contrast, certainty induces heuristic information processing, implying that agents base their judgements on superficial cues, such as records in stock indices. The latter case reinforces the view that, during phases of certainty shocks, regulators should take steps to counteract heuristic information processing.

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Appendix A Data

Table 2: Data descriptions, transformations, and sources

Variable	Symbol	Base Variable	Transformation	Code/Source
Real economic activity growth	Δq	real manufacturing industrial production	log differences	IP.B00004.S (FRB)
Real consumption growth	Δc	real personal consumption expenditures	log differences	DPCERC1 (BEA)
Inflation	Δp	personal consumption expenditure price deflator	log differences	DPCERG3 (BEA)
Change in interest rates	Δi	effective federal funds rate	differences	USFEDFUN (DS)
Stock returns	r	S&P 500	log differences	S&PCOMP (DS)
Stock market volatility	u_v	S&P 500 and since 1986 VXO index	-	see Bloom (2009)
Forecast dispersion in business outlook	u_{fd}	Philadelphia Fed's Business Outlook Survey	-	see Bachmann et al. (2013)
JLN macroeconomic uncertainty ($h = 1$ and 3)	u_m	-	-	see Jurado et al. (2015)
LMN financial uncertainty ($h = 1$ and 3)	u_f	-	-	see Ludvigson et al. (2018)

Notes: FRB denotes Federal Reserve Board, BEA Bureau of Economic Analysis, and DS Datastream. All data are seasonally adjusted when necessary.

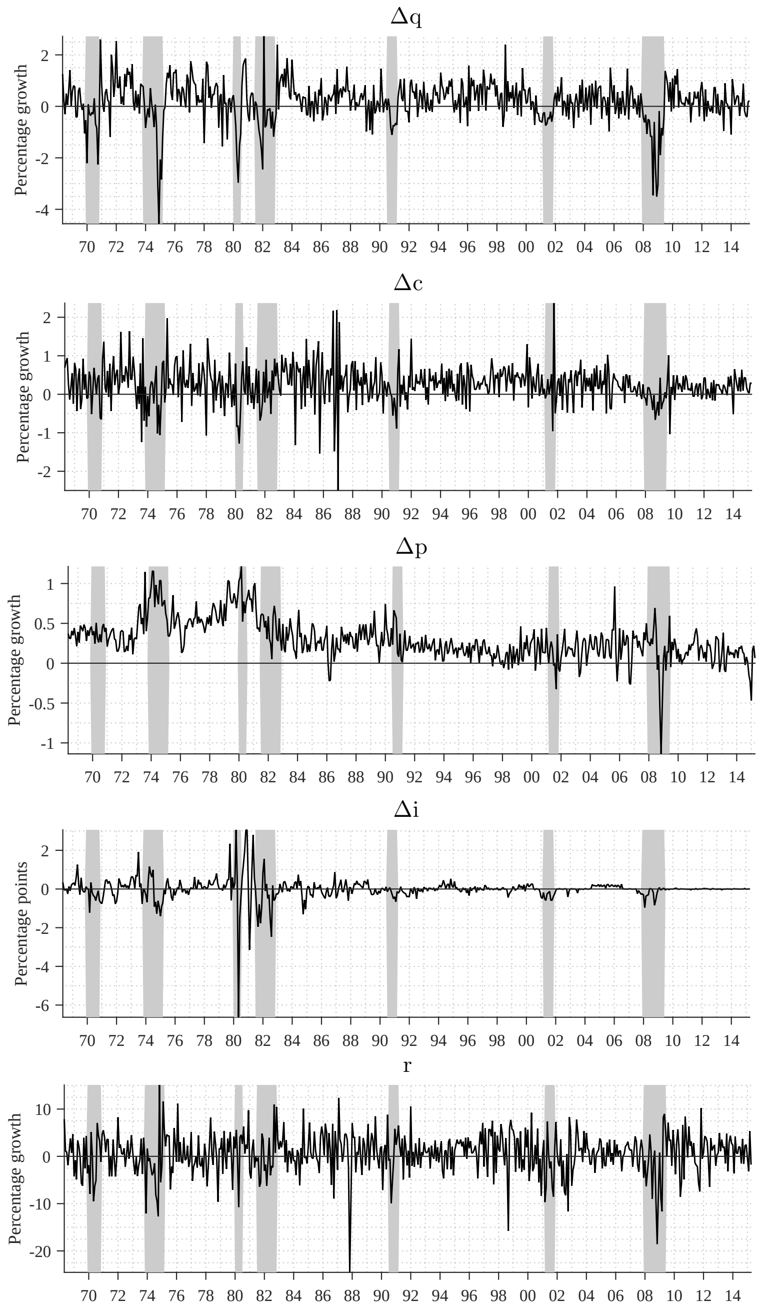

Figure 8: Time series of variables

Notes: Grey shaded areas mark NBER recession periods.

Appendix B Constructing the uncertainty-to-certainty ratio

This section summarizes the construction of the uncertainty-to-certainty ratio that closely relates to the construction of the indices by Baker et al. (2016).

Let x_t^{UC} and x_t^{C} denote the number of newspapers related to uncertainty and certainty in month t respectively. The uncertainty-to-certainty ratio (X_t) is the ratio of the two:

$$X_t = \frac{x_t^{\mathrm{UC}}}{x_t^{\mathrm{C}}} \cdot 100.$$

I use Factiva to determine the number of newspaper articles by considering ten different U.S. newspapers. These newspapers are: Los Angeles Times, USA Today, Chicago Tribune, Washington Post, Boston Globe, Wall Street Journal, Miami Herald, Dallas Morning News, Houston Chronicle, and San Francisco Chronicle.

The search phrase for x_t^{UC} is: (economy OR economic) AND ("stock market" OR "stock markets" OR "stock index" OR "stock indexes" OR "stock indices" OR S&P OR "Standard & Poor") AND (uncertainty OR uncertain OR fear OR concern OR panic OR worry OR doubt OR low) NOT (certain OR certainty OR trust OR faith OR confidence OR euphoria OR hope OR high).

The search phrase for x_t^{C} is: (economy OR economic) AND ("stock market" OR "stock markets" OR "stock index" OR "stock indexes" OR "stock indices" OR S&P OR "Standard & Poor") AND (certain OR certainty OR trust OR faith OR confidence OR euphoria OR hope OR high) NOT (uncertainty OR uncertain OR fear OR concern OR panic OR worry OR doubt OR low).

I deviate from Baker et al. (2016) in that I specifically focus on news related to stock markets and, furthermore, require that specific words do not appear.

Appendix C Newspaper headlines associated with the uncertainty and certainty shocks

Tables 3 to 7 show events that I argue are associated with exceedances of the uncertainty/certainty shock series. Specifically, I focus on first exceedances. First exceedances are exceedances that are not preceded by another exceedance (left-hand tail or right-hand tail) in the month before.

All tables refer to a representative newspaper article from the month indicated. I include additional keywords to describe the events in case the headline is not sufficiently informative about the underlying event. I classify the type of event into several categories: economic, environmental, oil, political, terror, and war.

I search for newspaper headlines using the New York Times online archives and Factiva (see Appendix B). Among other things, I use the The New York Times online archives to cover the time period prior to 1985. The search phrase is “Dow Jones” OR “Standard and Poor*” OR “Stock Market”, so as to filter news related to stock markets, in line with the identification of shocks.

Table 3: Uncertainty shocks

Headline (date), *newspaper* (or Bloom (2009))	*Additional keywords*	Date of shock	Type
Stocks slip to a '69 low (1969, July 10), *The New York Times*	*Lack of peace progress in Vietnam; growing tension in Middle East; concerned about widely predicted slowdown*	1969-07	War, Economic
Cambodia and Kent State (Bloom (2009))		1970-05	War
Stock prices drop across wide front (1971, August 4), *The New York Times*	*Nixon's Wage and Price Controls*	1971-08	Economic
Dow Industrials off 11.24 to 814.91, lowest of '71 (1971, November 12), *The New York Times*	*New pricing standards*	1971-11	Economic
Bewildered market skids 16.85 points (1973, February 15), *The New York Times*	*Dollar devaluation; Most massive decline in nearly 20 months*	1973-02	Economic
What price Watergate?; Crisis of confidence depresses markets, but it may be consequences, not cause (1973, May 23), *The New York Times*		1973-05	Economic, Political
Wide uncertainty on Nixon and oil stirs decline (1973, November 6), *The New York Times*	*Uncertainty over political future of President Nixon; oil shortages induced by the Middle East conflict*	1973-11	Political, Oil
Stocks retreat on rate fears (1982, January 26), *The New York Times*	*Renewed fears of rising interest rates*	1982-01	Economic
Dark days on Wall Street (1982, August 12), *The New York Times*; monetary cycle turning point (first volatility: Bloom (2009))	*Cruel economic news from all sides*	1982-08	Economic
Monetary cycle turning point (Bloom (2009))		1982-10	Economic
Foreign debt worries send Dow down by 10.21 (1983, July 8), *The New York Times*; Rate fear and earnings (1983, July 11), *The New York Times*	*Fear of rising interest rates; disappointing earnings*	1983-07	Economic
Stocks fall broadly, Dow off 8.48 (1984, January 31), *The New York Times*	*Concerns over direction of interest rates, the slowing economy and the huge Federal budget deficit*	1984-01	Economic
Industrials fall to 15 1/2-month low on worry about third world debt (1984, June 15), *The Wall Street Journal*	*Concern about repayment of debt*	1984-06	Economic
Economic indicators fall 0.8% (1984, August 30), *The Washington Post*	*Indicators declined for two consecutive months*	1984-08	Economic
International tensions drive market lower (1986, March 26), *Los Angeles Times*	*Libya conflict*	1986-03	War
Stock prices fall by record amount in busiest session (1986, September 12), *The New York Times*	*Growing concern about the nation's economy; worries about interest rates*	1986-09	Economic
Markets battered again; Dow off 34 (1987, April 15), *The Washington Post*	*Continuing fall of the dollar; persistent U.S. trade deficit*	1987-04	Economic
Black Monday (first volatility: Bloom (2009))		1987-10	Economic

Note: Bloom's (2009) dates of uncertainty refer to the maximum volatility dates unless otherwise indicated.

Table 4: Uncertainty shocks (continued)

Headline (date), *newspaper* (or Bloom (2009))	*Additional keywords*	Date of shock	Type
Dow falls 190 (1989, October 14), *Los Angeles Times*	*Fears that takeover fever is cooling and that record prices will moderate; Drop is worst since '87 crash stock market*	1989-10	Economic
Basic correction: unlike October dives, this stock market fall is due to fundamentals — as profits fall and rates rise many buyers stand aside and look for the bottom — a 250-point drop in 3 weeks (1990, January 26), *The Wall Street Journal*		1990-01	Economic
Middle East woes batter markets Dow plunges on surge in anxiety (1990, August 7), Chicago Tribune	*Tensions in Middle East; oil prices continued to skyrocket*	1990-08	War, Oil
The Dow's sell-off could signal a correction (1992, October 5), *The Wall Street Journal*	*Sick economy; election growing more uncertain; economic turmoil abroad among biggest trading partners*	1992-10	Economic, Political
Fed's move jolts stock and bond markets (1994, February 5) *The New York Times*	*Monetary cycle turning point; first increase in short-term interest rates in five years*	1994-02	Economic
Stock indexes slide to six-month lows markets (1996, July 24), *Los Angeles Times*; Stock markets skid on worry about profits, interest rates (1996, July 16), *The Washington Post*	*Technology sector leads retreat amid concerns that the pace of business will falter during the remainder of the year; stock market took one of its sharpest dives ever*	1996-07	Economic
Broad stock sell-off signals change in market — cyclical issues take command (1997, August 18), *The Wall Street Journal*	*Weaker dollar; rising interest rates; outlook of lower-than-expected earnings; second largest decline ever of Dow*	1997-08	Economic
Asian Crisis (Bloom (2009))		1997-11	Economic
Russian, LTCM default (Bloom (2009))		1998-08	Economic
Crisis is deepening in Brazil markets (1999, January 15), *The New York Times*	*Brazil's economic crisis*	1999-01	Economic
Greenspan's remarks send down 108; yields jump (1999, August 28), *Los Angeles Times*; When the bubble bursts... (1999, August 18), *The Wall Street Journal*	*Concerns over the highly valued stock market*	1999-08	Economic
Markets shaken as economic statistics fan inflation fears (2000, January 29), *The New York Times*	*Rising short-term rates and fear over interest rate rise; Three major market indexes prepared to post their worst performance for January in a decade or more;*	2000-01	Economic
Stock market in steep drop as worried investors flee; Nasdaq has its worst week (2000, April 15), *The New York Times*	*One of the worst weeks in the history of United States markets; higher-than-expected inflation*	2000-04	Economic

Note: Bloom's (2009) dates of uncertainty refer to the maximum volatility dates unless otherwise indicated.

Table 5: Uncertainty shocks (continued)

Headline (date), *newspaper* (or Bloom (2009))	*Additional keywords*	Date of shock	Type
Volatile world events cause investor flight from stocks (2000, October 13), *The New York Times*	*Oil prices surging; turmoil in middle east escalating*	2000-10	War, Oil
Markets plunge in wide sell-off; Nasdaq falls 6% (2001, March 13), *The New York Times*	*Worries about slowing economy; declining corporate earnings*	2001-03	Economic
9/11 terrorist attack (Bloom (2009)))		2001-09	Terror
Year's first half is worst in 32 years; June 24-28, 2002 (2002, June 30) *The Washington Post*	*Economic conditions; corporate accounting scandals*	2002-06	Economic
Worldcom Enron (Bloom (2009))		2002-09	Economic
Gulf War II (Bloom (2009))		2003-02	War
Asia and Europe stocks follow Wall Street (2007, March 14), *The New York Times*	*Concerns spread about the consequences of loose lending practices in the United States housing market*	2007-03	Economic
Impact of mortgage crisis spreads — Dow tumbles 2.8% as fallout intensifies; moves by central banks (2007, August 10), *The Wall Street Journal* (Credit crunch: first volatility: Bloom (2009))	*BNP Paribas decides to suspend three hedge funds focused on US mortgages*	2007-08	Economic
It's official. Wall Street correction — industrials, S&P 500 drop 10% from highs as recession fears grow (2007, November 27), *The Wall Street Journal*		2007-11	Economic
Global stocks plunge as U.S. crisis spreads; sell-offs in all major exchanges (2008, January 22), *The Washington Post*		2008-01	Economic
Lehman Brothers' collapse (Bloom (2009))		2008-09	Economic
Stocks plunge on fears of a spreading European crisis (2010, May 21), *The New York Times*		2010-05	Economic
Shares fall amid concerns about Japan (2011, March 13), *The New York Times*	*Nuclear crisis in Japan*	2011-03	Environmental
Stock market plummets after historic downgrade of U.S. credit rating (2011, August 9), *The Washington Post*	*S&P downgrades U.S. credit rating*	2011-08	Economic
Markets extend slide over Fed concerns, poor economic news from China (2013, June 21), *The Washington Post*	*Investors are anxious the Fed will pull back on stimulus and unnerved by weak economic data from China; biggest one-day drop since 2011*	2013-06	Economic
Steep sell-off spreads fear to Wall Street (2014, October 16), *The New York Times*	*Fear that governments and central banks have failed to anticipate the recent weakening in the global economy; particularly in Europe; Vix to its highest level since 2011*	2014-10	Economic

Note: Bloom's (2009) dates of uncertainty refer to the maximum volatility dates unless otherwise indicated.

Table 6: Certainty shocks

Headline (date), *newspaper*	*Additional keywords*	Date of shock	Type
Market rallies on broad front (1968, August 13), *The New York Times*	*Peace hopes for Vietnam*	1968-08	War
Dow is up by 11.59 in heavy trading (1972, February 10), *The New York Times*	*Best level since early September*	1972-02	Economic
Stocks rebound on a broad front (1972, June 15), *The New York Times*	*Partial recovery after a prolonged decline touched off by profit taking; "technical rebound"; there was no hard news to account for the snapback*	1972-06	Economic
The Dow at 1,000 (1972, November 17), *The New York Times*	*Above 1,000 for the first time in history*	1972-11	Economic
Oil-price optimisim lifts market (1976, December 15), *The New York Times*	*Saudi Arabian oil minister had called for six-month freeze in oil prices*	1976-12	Oil
Stock prices climb briskly (1981, January 28), *The New York Times*	*Prospects for early decontrol of domestic crude oil price*	1981-01	Oil
Dow jumps to 1,070.55, a record (1982, December 28), *The New York Times*	*Signs of an economic recovery*	1982-12	Economic
Stocks gain in late rally; 2 indexes shatter records (1985, April 26), *Chicago Tribune*		1985-04	Economic
Stocks indexes end at record levels after early dip on bond weakness (1986, May 30), *The Wall Street Journal*		1986-05	Economic
Stocks rocket to 3rd straight record high (1987, July 31), *Houston Chronicle*		1987-07	Economic
Dow finishes year above 2,000 mark (1988, December 31), *Houston Chronicle*	*Bluechip (...) at highest levels since October 1987 crash*	1988-12	Economic
Dow industrials hit record on upbeat economic news (1993, October 29), *The Wall Street Journal*	*Increase in third-quarter GDP exceeded analysts' expectations*	1993-10	Economic
Dow index climbs 35 to yet another record; The Standard & Poor's 500 index also hits a new high (1993, December 28), *San Francisco Chronicle*; Industrials reach record again as recovery signs ignite cyclicals (1993, December 14), *The Wall Street Journal*		1993-12	Economic
Binge of stock buybacks makes 1994 a record year (1994, December 19), *The Wall Street Journal*	*Buybacks are important signal to investors*	1994-12	Economic
Dow industrials close above 5000 mark (1995, November 22), *The Wall Street Journal*	*Just nine months after crossing the 4000 barrier*	1995-11	Economic
The S.&P. 500 breaks through the 1,000 mark (1998, February 3), *The New York Times*; Dow industrials jump 115.09, back to a record (1998, February 11), *The Wall Street Journal*		1998-02	Economic
What correction? With dazzling speed, market roars back to another new high — surge puts the Dow at 9374 in a lightning reversal of autumn's doldrums — 'Nothing to get in its way' (1998, November 24), *The Wall Street Journal*	*Widespread euphoria*	1998-11	Economic

Table 7: Certainty shocks (continued)

Headline (date), *newspaper*	*Additional keywords*	Date of shock	Type
Stocks rally, though tentative, offers hope for economy (2003, April 24), *Los Angeles Times*	*War in Iraq has wound down*	2003-04	Economic
Treasuries stage sharp rally on consumer price data (2004, June 16), *The New York Times*	*Stocks also rose; the market was priced for accelerating inflation*	2004-06	Economic
As Bush goes, so goes market — major indexes are up in month that historically is the weakest, echoing bets on a re-election (2004, September 20), *The Wall Street Journal*		2004-09	Political
Stocks advanced last week, ... (2005, February 28), *The Washington Post*	*"...powered by an upbeat report on economic growth and some consumer inflation data that didn't rattle investors."*	2005-02	Economic
Shares rally after crude oil prices decline $2 a barrel (2005, June 29), *The New York Times*	*Consumer confidence jumped to a three-year high*	2005-06	Oil
Dow Jones passes 12,000 for the first time (2006, October 18), *The New York Times –International Harald Tribune*		2006-10	Economic
Shares rise, erasing Dow's loss for '09 (2009, June 13), *The New York Times*; Economic data push the Dow 2.6% higher (2009, June 2), *The New York Times*	*S.&.P 500-index best level in five months; signs of economic growth in China; stability in Europe; sings of improvement in construction and manufacturing in the United States*	2009-06	Economic
Stocks soar, but many ask why (2010, March 29), *The New York Times*		2010-03	Economic
S.&P. 500 reaches 2-year high as shares post modest gains (2010, December 21), *The New York Times*		2010-12	Economic
Stocks hit 5-month high in year-end rebound (2011, December 24), *The Wall Street Journal*	*Accelerating recovery; break in the latest congressional deadlock*	2011-12	Economic
Markets rally as volatility hits five-year low (2012, August 18), *The Washington Post*	*S&P 500 index near four-year high*	2012-08	Economic
Stocks near record highs; The S&P 500 rises for the eigth day, closing above 1,500 for the first time since 2007 (2013, January 26), *Los Angeles Times*		2013-01	Economic
S.&P. index surpasses high point of 2007 (2013, March 29), *The New York Times*		2013-03	Economic
S&P rises into the spotlight — broad index tops 1800 for the first time a day after DJIA climbs past 16000 (2013, November 23), *The Wall Street Journal*		2013-11	Economic
S.&P. 500-stock index closes at new high (2014, May 24), *The New York Times*	*Better-than-expected home sales*	2014-05	Economic

Appendix D Other proxies of uncertainty

The set of other proxies of uncertainty includes a measure of forecast dispersion that exploits the Philadelphia Fed's Business Outlook Survey as proposed by Bachmann et al. (2013) (u_{fd}). It is based on expectations of future developments in general business activity. The other two proxies are argued to capture fundamental financial and macroeconomic uncertainty, focussing on the time-varying variance of the unforecastable component of a large set of variables. They have been suggested by Ludvigson et al. (2018) (LMN, u_f) and Jurado et al. (2015) (JLN, u_m).[24]

I use the BQVAR to analyze shocks identified through u_{fd}. In contrast, I use a regular VAR for identifying shocks to u_f and u_m. The reason is that both proxies already reflect the outcome of an empirical exercise and measure uncertainty at the conditional mean.

First, I present the results using Bachmann et al.'s (2013) proxy of uncertainty. Thereafter, I discuss my findings for u_f and u_m.

D.1 Forecast dispersion

Similar to the main analysis, I identify an exogenous uncertainty (certainty) shock to the conditional 0.9 (0.1) quantile of the proxy of uncertainty that may impact the conditional 0.1 (0.9) quantile of stock returns contemporaneously. I discuss the impact on the median of the real and policy sector variables. The PQIRF is shown in Figure 9 and the PFEVD in Table 8.

Overall, the findings suggest that an uncertainty shock identified through Bachmann et al.'s (2013) proxy of uncertainty does not have any non-linear impact on the real economy. A normalized shock to the conditional right-hand tail of u_{fd} leads to a decline in economic activity growth of -0.2 p.p one month after the shock. Similarly, a normalized shock to the conditional left-hand tail of u_{fd} leads to an initial rise in economic activity growth by 0.2 p.p. The PFEVD supports this finding as both shocks account for a similar percentage of fluctuations in the macroeconomic variables.

D.2 Fundamental financial and macroeconomic uncertainty within a regular VAR

The impulse responses are shown in Figure 10 and the forecast error variance decompositions in Table 8.

Overall, the impact of my uncertainty shock differs from the impact of uncertainty shocks identified through u_f and u_m. Most markedly, uncertainty shocks based on the latter proxies are much less important for the macroeconomic time series, but also for stock returns. For instance, the shocks account for 0.8% (u_f) and 3.2% (u_m) of fluctuations in real activity growth over the first half-year. Furthermore, the two identified shocks are relatively unimportant for stock markets (1.3% for u_f and 0.2% for u_m during the first half-year).

The dynamics of the two uncertainty shocks differ as well. On impact, u_f and u_m rise in response to a shock and decline only thereafter. In parallel, the two shocks lead a contemporaneous rise in stock returns.

[24]The authors propose several forecast horizons for their measures. In this study, I use the instantaneous indices, i.e., a forecast horizon of one, when comparing these indexes to my identified contemporaneous uncertainty shocks. The tree-month horizon indices u_f and u_m are considered in the robustness exercise, in which I identify uncertainty shocks from a VAR on the basis of these indicators. The authors also use the three-month horizon indices in their VAR analysis.

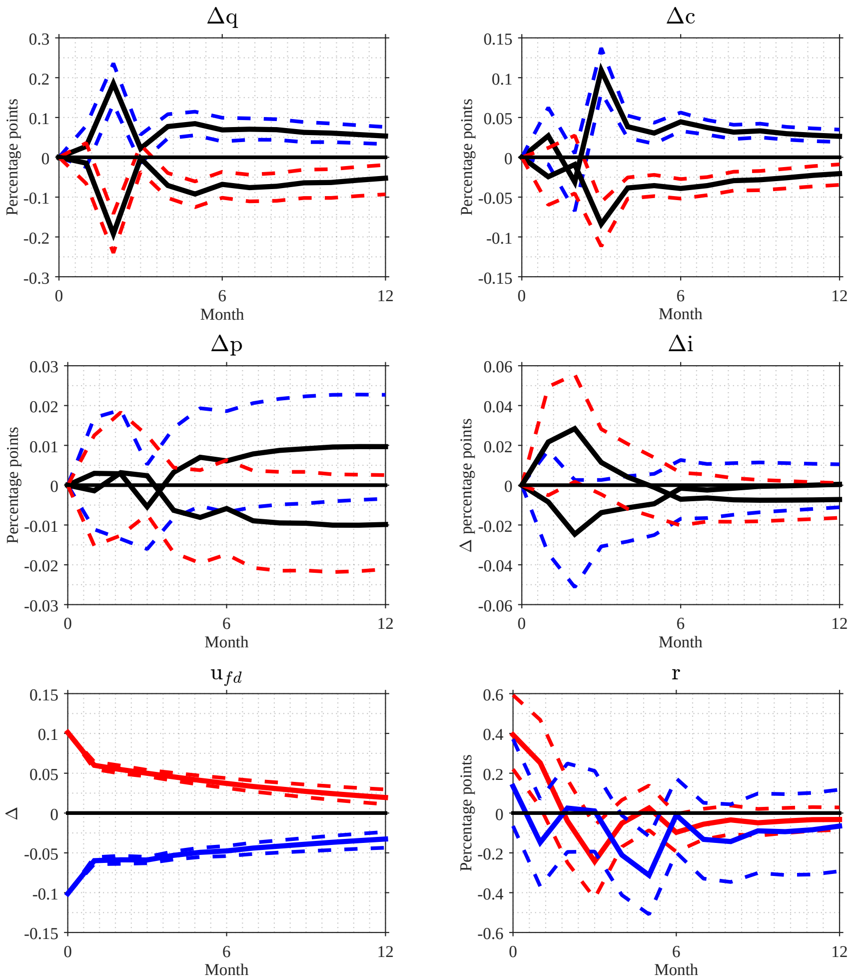

Figure 9: The impact of an u_{fd} uncertainty shock and an u_{fd} certainty shock on the conditional median of real and policy sector variables

Notes: The figures depict the pseudo-impulse responses of the conditional median of the macroeconomic and policy sector variables (Δq, Δc, Δp, Δi) and the conditional tails of u_{fd} and r in response to an u_{fd} uncertainty and an u_{fd} certainty shock. Specifically, I identify an u_{fd} uncertainty shock to the conditional 0.9 and 0.1 quantiles of u_{fd} and r respectively. I identify a certainty shock to the conditional 0.1 and 0.9 quantiles of u_{fd} and r respectively. The color blue (red) marks the conditional 0.1 (0.9) quantile. For more details on shock identification, see Section 3.3. Solid lines refer to the median of the distribution of the impulse responses and dashed lines correspond to posterior 68% probability bands. Blue dashed lines mark responses to a shock to the conditional left-hand tail of u_{fd} and red dashed lines to the conditional right-hand tail of u_{fd}. Δq denotes growth in industrial production, Δc is growth in consumption, Δp is inflation, Δi is changes in the interest rate, u_{fd} is the proxy of uncertainty (forecast dispersion), and r denotes stock returns.

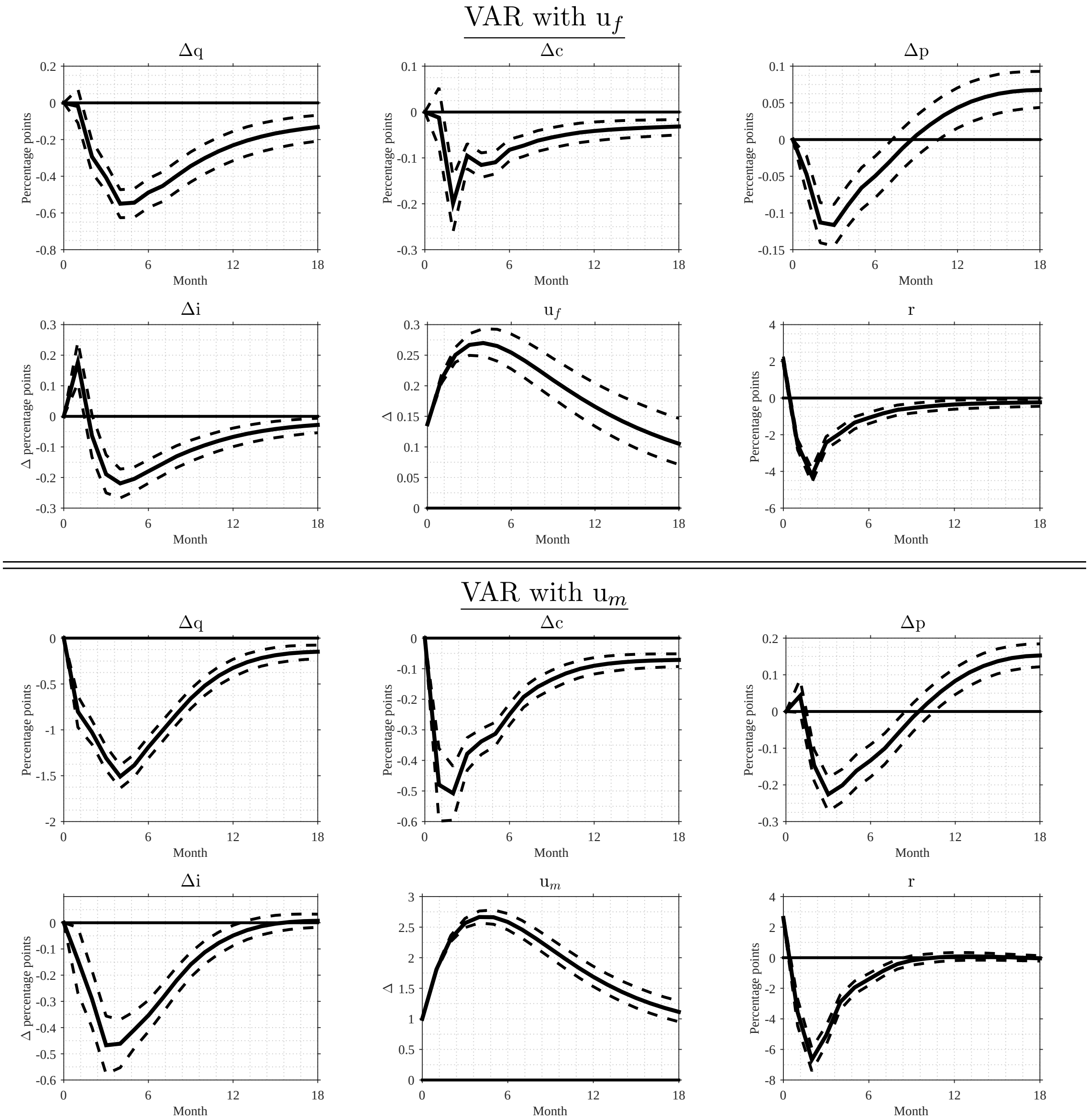

Figure 10: The impact of a u_f and a u_m uncertainty shock identified within a regular VAR

Notes: u_f is the measure of fundamental financial uncertainty proposed by Ludvigson et al. (2018). u_m is the measure of fundamental macroeconomic uncertainty introduced by Jurado et al. (2015). Solid lines refer to the median of the distribution of the impulse responses and dashed lines correspond to posterior 68% probability bands. Δq denotes growth in industrial production, Δc is growth in consumption, Δp is inflation, Δi is changes in the interest rate, $u_{\cdot}$ is the proxy of uncertainty, and r denotes stock returns.

Table 8: Variance explained by various shocks (in percent)

	u_{fd} uncertainty shock	u_{fd} certainty shock	u_f	u_m
Average 0-6 months				
Δq	2.9	3.2	0.8	3.2
Δc	1.3	2.2	0.2	0.9
Δp	0.1	0.1	0.5	0.7
Δi	1.0	0.8	0.9	1.3
$u_.$	88.8	92.0	95.8	94.0
r	0.1	0.3	1.3	0.2
Average 7-12 months				
Δq	5.4	6.9	2.8	8.1
Δc	3.2	5.0	0.4	1.7
Δp	0.5	0.4	0.8	1.4
Δi	1.5	1.2	2.6	3.7
$u_.$	64.7	73.2	92.1	84.6
r	0.3	0.4	2.1	0.4

Notes: The table shows the pseudo forecast error variance of a u_{fd} uncertainty shock and a u_{fd} certainty shock using the BQVAR for the conditional median of the real and policy sector variables (Δq, Δc, Δp, Δi) and the conditional tails of u_{fd} and r. Specifically, I identify a u_{fd} uncertainty shock to the conditional 0.9 and 0.1 quantiles of u_{fd} and r respectively. I identify a u_{fd} certainty shock to the conditional 0.1 and 0.9 quantiles of u_{fd} and r respectively. Furthermore, the table depicts the forecast error variance of shocks identified to the fundamental uncertainty indices u_f and u_m within a regular VAR framework. Δq denotes growth in industrial production, Δc is growth in consumption, Δp is inflation, Δi is changes in the interest rate, $u_.$ is the proxy of uncertainty, and r denotes stock returns. u_{fd} is Bachmann et al.'s (2013) proxy of uncertainty, which is a measure of forecast dispersion. u_f is the measure of fundamental financial uncertainty proposed by Ludvigson et al. (2018). u_m is the measure of fundamental macroeconomic uncertainty introduced by Jurado et al. (2015).

Appendix E Conditional tail shocks of SMV affecting the conditional median of stock returns

Table 9: Variance explained by conditional tail shocks on SMV that affect the conditional median of stock returns (in percent)

Variable	Δq	Δc	Δp	Δi	u_v	r
Quantile	0.5	0.5	0.5	0.5		0.5
Right-hand tail shock ($\tau_5 = 0.9$)						
Months 0–6	4.2	1.5	3.2	5.4	92.0	27.6
Months 7–12	9.1	1.9	3.2	5.4	72.9	30.6
Left-hand tail shock ($\tau_5 = 0.1$)						
Months 0–6	0.6	0.1	0.4	0.7	93.1	8.8
Months 7–12	1.0	0.1	0.4	1.1	86.9	10.6

Notes: The table shows the pseudo forecast error variance of a conditional right-hand and left-hand tail shock to u_v for the conditional median of Δq, Δc, Δp, Δi, and r. The figures refer to an average of the pseudo forecast error variance over the months indicated. Δq denotes growth in industrial production, Δc is growth in consumption, Δp is inflation, Δi is changes in the interest rate, u_v is the proxy of uncertainty (stock market volatility), and r denotes stock returns.

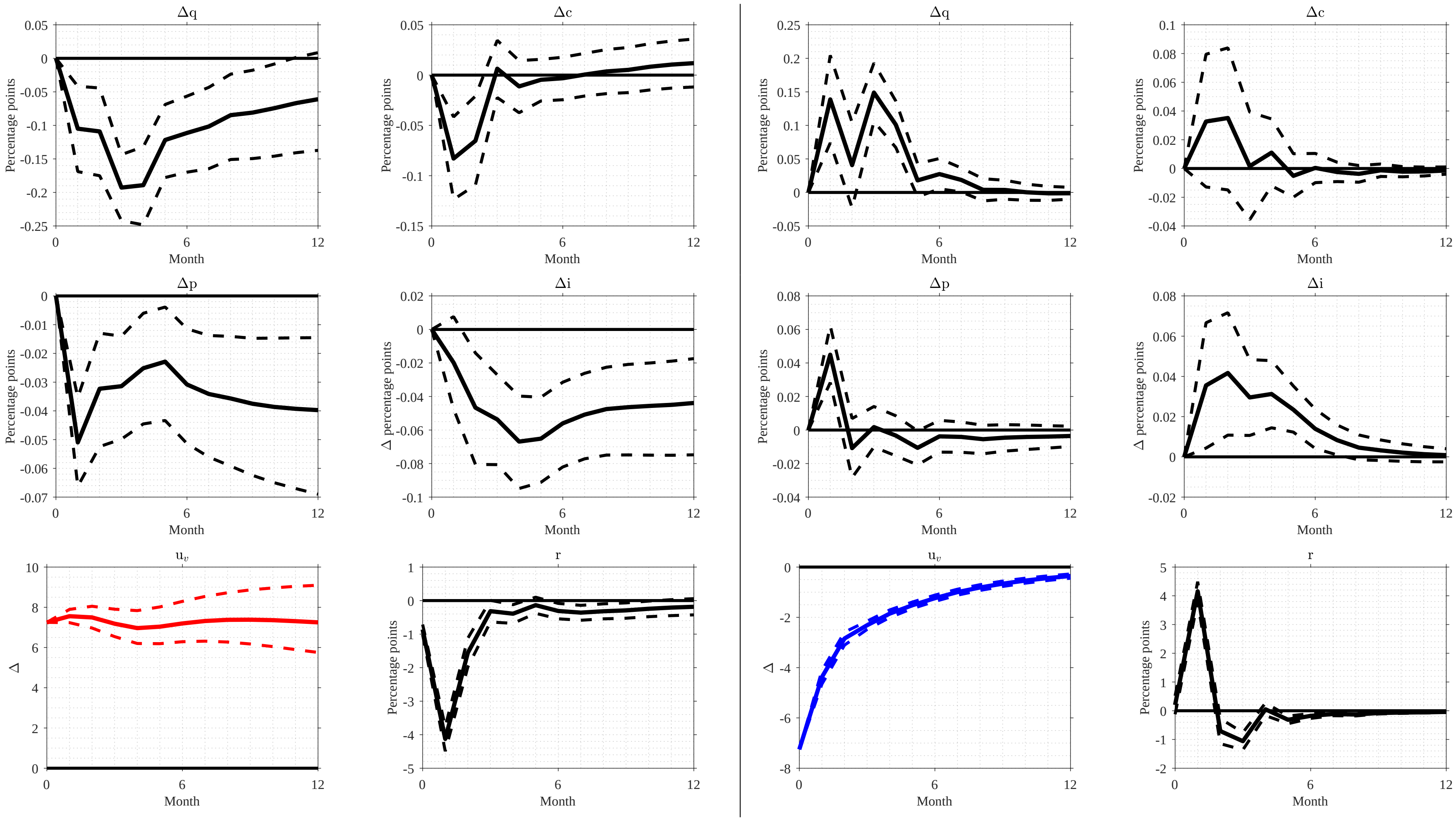

Figure 11: The impact of conditional right-hand (left panel) and left-hand (right panel) tail shocks of SMV that affect the conditional median of stock returns

Notes: The panels depict the pseudo-impulse responses of the conditional median of Δq, Δc, Δp, Δi and r in response to a conditional right-hand and left-hand tail shock identified through u_v. Solid lines refer to the median of the distribution of the impulse responses and the dashed lines correspond to posterior 68% probability bands. Δq denotes growth in industrial production, Δc is growth in consumption, Δp is inflation, Δi is changes in the interest rate, u_v is the proxy of uncertainty (stock market volatility), and r denotes stock returns.

Appendix F Tail shocks identified within a recursive structural QVAR

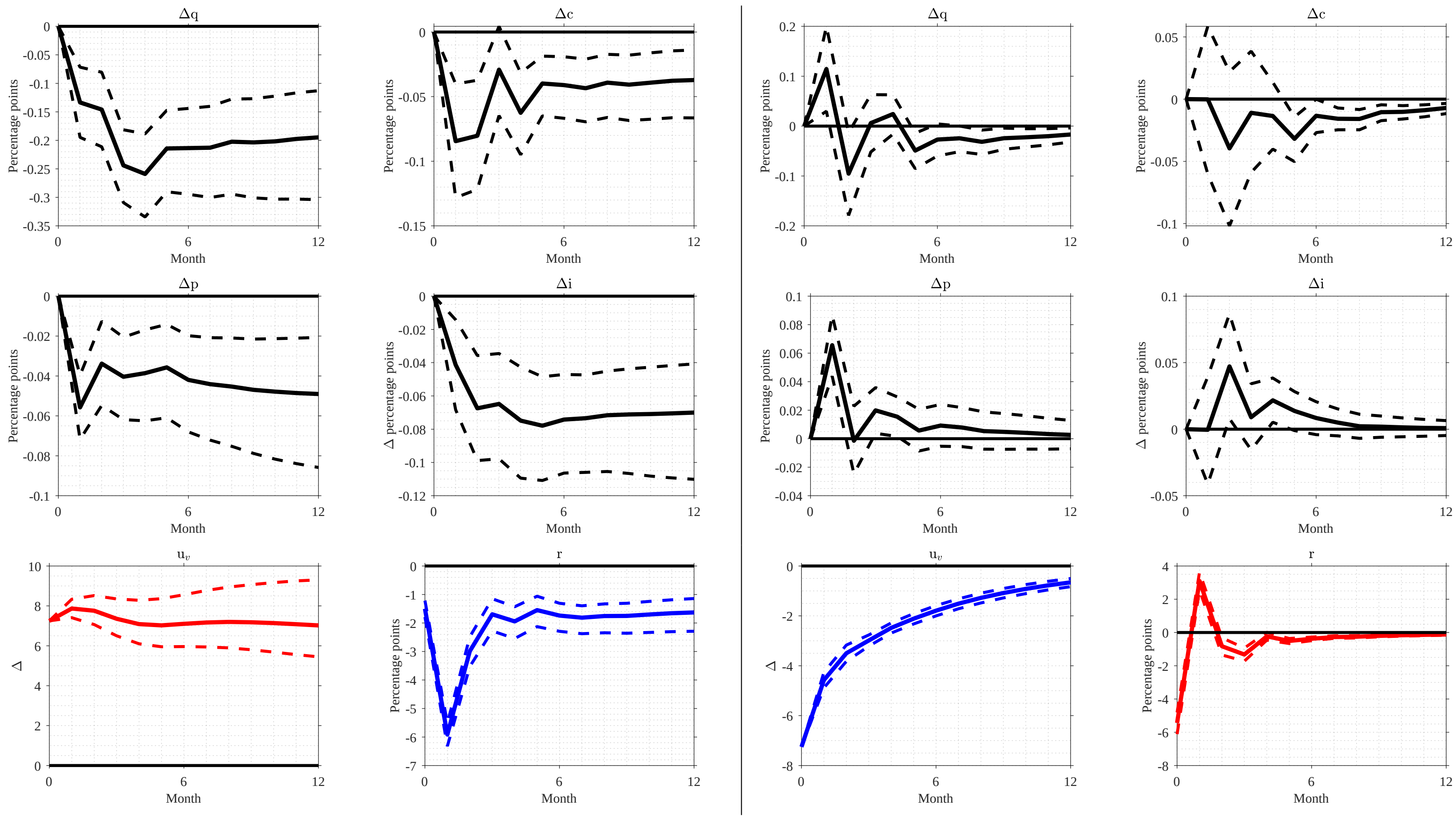

Figure 12: The impact of an uncertainty shock (left panel) and a certainty shock (right panel) identified within a recursive structural QVAR

Notes: Using a recursive structural QVAR, as suggested by Chavleishvili and Manganelli (2019), the panels depict the pseudo-impulse responses of the conditional median of the macroeconomic and policy sector variables (Δq, Δc, Δp, Δi) and the conditional tails of the financial sector variables (u_v, r) in response to an uncertainty and a certainty shock. Specifically, I identify an uncertainty shock to the conditional 0.9 and 0.1 quantiles of u_v and r respectively. I identify a certainty shock to the conditional 0.1 and 0.9 quantiles of u_v and r respectively. The color blue (red) marks the conditional 0.1 (0.9) quantile. For more details on shock identification, see Section 3.3. Solid lines refer to the median of the distribution of the impulse responses and dashed lines correspond to posterior 68% probability bands. Δq denotes growth in industrial production, Δc is growth in consumption, Δp is inflation, Δi is changes in the interest rate, u_v is the proxy of uncertainty (stock market volatility), and r denotes stock returns.